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GROUP VELOCITY AND DISPERSION COEFFICIENT IN THE DISTRIBUTED SENSOR OF VIBRATION IN FIBRE OPTIC MICHELSON'S INTERFEROMETER CONFIGURATION

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Abstract: *The problem of determination of group velocity and dispersion coefficient in the distributed sensor of vibration in fibre optic Michelson's interferometer configuration is considered in the report. Influence of acoustic wave on optical wave in a Bragg modulator is described. It is shown that such interferometer can be used as a distributed highly sensitive sensor of mechanical values (vibration, acoustic wave, pressure, displacement).*

Keywords: – *Michelson's interferometer, fibre optic, distributed vibration sensor*

1. INTRODUCTION

Interferometer as a classical technique is known for over 100 years. First research in this discipline were conducted by Michelson (1881), Mach – Zehnder (1891), Fabry and Perot (1899) [1]. Fiber – optic interferometer has been developing for the last ten years as a separate sort of fiber – optic sensors, functioning on a principle of optical wave phase changes detection. Optical configuration of interferometers has been adapted from classical interferometers where the light is lead in optical wave duct [2].

In scientific literature interferometric fiber – optics sensor is described as a device that processed of substitution of optical wave physical parameters for electric signal. Optic fibre sensor can be applied to detect a wide variety of external signals. The influence of mechanical (vibration, acoustic wave [3], pressure, displacement), magnetic [4], electric fields or temperature on light wave propagation can be easily detected event in closed optics circuit. Optic fibre interference sensors have extremely high sensitivity and closed optic circuit is their another advantage.

Optic fibre sensor consists of a light source, optic fibre and light detector, giving electric output signal. Optic fibre circuit has a sensing element – phase converter, which changes the phase of a light wave according to external signal. Those changes are converted back to electric signal in demodulator, which is an integral part of detector unit [5].

2. WAVES IN THE FIBRE OPTIC MICHELSON'S INTERFEROMETER CONFIGURATION

Single mode optic fibre carries coherent light wave, which has properties similar to a flat wave and is capable of interference. Distributed fibre optic sensor in Michelson interferometer layout is presented on Fig 1. If both arm of this interferometer are equal, it is called balanced one.

The phase distortion along the path L1 can be obtained on the basis of interference waves reflected from both arm of a sensor. Additional phase shift is introduced to achieve a clear distinction between consecutive wave packets.

If lengths of interferometer arms are different it is called unbalanced one. A Bragg cell (Fig 2) works as a light frequency shifter.

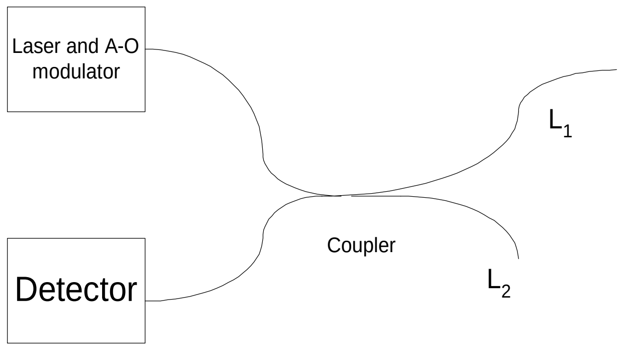

Fig. 1 - Unbalanced Michelson's interferometer as a distributed sensor.

In Fig. 2:

Θ_P – angle of incidence of optical wave at Bragg modulator,

Θ_P – Bragg angle of refraction after passing the modulator,

ω_i – angular frequency of optical wave modulated by acoustic wave,

Ω_i – angular frequency of acoustic wave,

L_B – Bragg modulator width,

$\mathbf{k}$ – optical wave vector,

λ – optical wave length,

$\mathbf{K}$ – acoustic wave vector,

Λ – acoustic wave length,

$\mathbf{V}_a$ – acoustic wave velocity,

$\mathbf{k}_0$ – optical wave that falls on a Bragg modulator,

$\mathbf{k}_i$ – optical wave after a Bragg modulator.

Interferometer is excited with wave packets shorter than their propagation time through the arm. The delay between packets τ is accordingly set to obtain interference in the coupler:

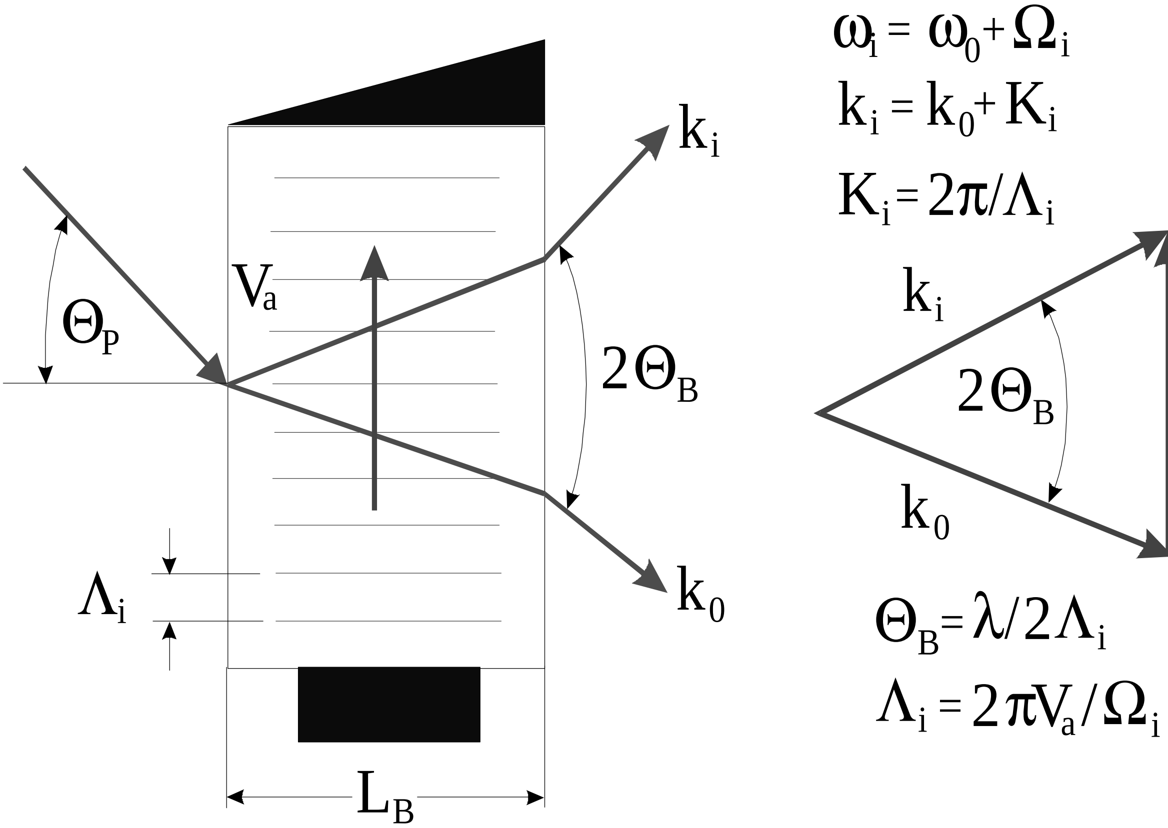

Fig 2 - Description of light frequency modulation with acoustooptic effect.

$$\tau = \frac{2L}{v}, \qquad (1)$$

where

$L = L_1 - L_2$ – difference of lengths of the interferometer arms;

V – speed of light in optic fibre.

As a result, a signal proportional to phase difference is obtained, having $\Delta f = f_1 - f_2$ frequency.

To simplify the analysis, polarization effects has been neglected which means, that constant polarization is assumed. Light wave at the output of frequency shifter (Fig. 2) with wave vector $\mathbf{k}_i = \mathbf{k}_0 + \mathbf{K}_i$ and frequency $\omega_i = \omega_0 + \Omega_i$ can be described as

$$E = \frac{1}{2} U_B \exp[j(\Phi + \omega_i t - \mathbf{k}_i \mathbf{r})], \qquad (2)$$

where

$$U_B = U_p \sin\frac{\Theta}{2}, \qquad (3)$$

where

U_p – amplitude of incident wave,

Θ – Bragg angle,

$\mathbf{\Phi}$ – initial phase.

To simplify the description the following assumption were made: optic coupling Bragg cell – fibre is lossless, wave is divided with 1:2 ratio and wave returning to coupler after passing measurement arm can be described as:

$$E = \frac{1}{4} t_{m1} U_B e^{j(\Phi + \omega_1 t - \beta L_1)} \qquad (4)$$

where

t_{m1} – transmission coefficient.

3. GROUP VELOCITY AND DISPERSION COEFFICIENT IN DISTRIBUTED SENSOR OF VIBRATION

A light wave modulated by a Bragg modulator assumes the following form (using its electrical vector):

$$E = U_B \exp[j(\Phi + \omega t - \beta L)], \qquad (5)$$

where

U_B – amplitude of the wave modulated in a Bragg modulator,

ω – angular frequency of light wave,

β – propagation constant of light wave in light guide,

L – light guide length.

Propagation constant of light wave in light guide can be presented as

$$\beta = n(\mathbf{K} + \mathbf{k})(1 - \Delta \mathrm{B}) \qquad (6)$$

where

n – index of refraction of light guide core,

K – wave vector of the acoustic wave,

k – wave vector of the light wave,

Δ, B – light guide parameters;

β - light guide propagation constant expressed by the ratio

$$\beta = \mathbf{n}_1 \left|(\mathbf{K} + \mathbf{k})\right| (1 - \Delta_1 \mathbf{B}),$$

where

$$\Delta_1 = \frac{\mathbf{n}_1 - \mathbf{n}_2}{\mathbf{n}_1},$$

$$\mathbf{B} = \frac{\mathbf{n}_1^2 \mathbf{k}_0^2 - \beta^2}{\mathbf{n}_1^2 \mathbf{k}_0^2 - \mathbf{n}_2^2 \mathbf{k}_0^2} = \frac{\mathbf{n}_1^2 - \mathbf{n}_{ef}^2}{\mathbf{n}_1^2 - \mathbf{n}_2^2},$$

$$\mathbf{n}_{ef} = \frac{\beta}{\mathbf{k}_0}.$$

Light wave falls at Bragg angle Θ at the acoustic-optical modulator. Using Cartesian coordinate system to the modulator location in such way that the Y axis is directed along acoustic wave propagation direction, the components of the two vectors in this system will have the following form:

$$\mathbf{K}\!: [0, \mathrm{K}],$$

$$\mathbf{k}\!: [\mathrm{k}_0 \cos(\Theta), \mathrm{k}_0 \sin(\Theta)] \qquad (7)$$

Module of the stochastic vector is expressed by the following formula:

$$\left|\mathbf{K_w}\right| = \sqrt{[\mathbf{k_0}\cos(\Theta)]^2 + [\mathbf{K} + \mathbf{k_0}\sin(\Theta)]^2} =$$

$$= \sqrt{(\mathbf{k_0})^2 + (\mathbf{K})^2 + 2\mathbf{k_0}\mathbf{K}\sin(\Theta)}; \qquad (8)$$

from (8) follows that

$$\left|\mathbf{K_w}\right| = \mathbf{k_0}\sqrt{1 + \left(\frac{\mathbf{K}}{\mathbf{k_0}}\right)^2 + 2\frac{\mathbf{K}}{\mathbf{k_0}}\sin(\Theta)} \qquad (9)$$

Expanding expression (9) into Taylor's series we receive in the first approximation:

$$\left|\mathbf{K_w}\right| \approx \mathbf{k_0}[1 + \frac{1}{2}(2\frac{\mathbf{K}}{\mathbf{k_0}}\sin(\Theta) + \left(\frac{\mathbf{K}}{\mathbf{k_0}}\right)^2 \cos^2(\Theta)) + ...] \qquad (10)$$

Substituting expression (10) into (6), the formula for propagation constant of light wave modulated acoustically wave can be obtained:

$$\beta \approx \mathbf{n}\mathbf{k}_0[1 + \frac{1}{2}(2\frac{\mathbf{K}}{\mathbf{k_0}}\sin(\Theta) +$$

$$+ \left(\frac{\mathbf{K}}{\mathbf{k_0}}\right)^2 \cos^2(\Theta))](1 - \Delta\mathbf{B}). \qquad (11)$$

Wave vectors depend on wave length:

$$\mathbf{k}_0 = \frac{2\pi}{\lambda_0}$$

for light wave with wave length λ_0,

$$\mathbf{K} = \frac{2\pi}{\Lambda}$$

for light wave with wave length Λ, and

$$\mathbf{c} = \lambda_0 \nu_0,$$

$$\mathbf{v} = \Lambda \mathbf{f},$$

$$\mathbf{k}_0 = \frac{2\pi\nu_0}{\mathbf{c}},$$

$$\mathbf{K} = \frac{2\pi\mathbf{f}}{\mathbf{v}},$$

v – acoustic wave velocity,

$$\beta \approx [\mathbf{n}\mathbf{k}_0 + \frac{1}{2}2\frac{2\pi\mathbf{n}}{\mathbf{v}}\mathbf{f}\sin(\Theta) +$$

$$+\frac{1}{2}\mathbf{f}^2\frac{2\pi\mathbf{nc}}{\nu_0\mathbf{v}^2}\cos^2(\Theta)](1-\Delta\mathbf{B}),$$

$$\beta \approx 2\pi\mathbf{n}[\frac{\nu_0}{\mathbf{c}} + \mathbf{f}\frac{\sin(\Theta)}{\mathbf{v}} + \frac{1}{2}\mathbf{f}^2\frac{\mathbf{c}\cos^2(\Theta)}{\nu_0\mathbf{v}^2}](1-\Delta\mathbf{B}) \quad (12)$$

Assuming that dependency of propagation constant $\beta(\nu_0+f)$ expanded into Taylor's series is expressed by

$$\beta(\nu_0+f) = \beta(\nu_0) + f\frac{d\beta}{d\nu} + \frac{1}{2}f^2\frac{d^2\beta}{d\nu^2}, \quad (13)$$

group velocity is equal to

$$\frac{1}{v_{gr}} = \frac{1}{2\pi}\frac{d\beta}{d\nu},$$

$$\frac{1}{\mathbf{v_{gr}}} = \frac{n\sin(\Theta)}{\mathbf{v}}(1-\Delta\mathbf{B}), \quad (14)$$

and dispersion coefficient equals

$$D_\nu = \frac{1}{2\pi}\frac{d^2\beta}{d\nu^2},$$

$$\mathbf{D}_\nu = \frac{\mathbf{nc}\cos^2(\Theta)}{\nu_0\mathbf{v}^2}(1-\Delta\mathbf{B}). \quad (15)$$

4. CONCLUSIONS

Group Velocity

If the dispersion coefficient is sufficiently small, the third term in the expansion (13) may be neglected and

$$E(f)=E(0)\exp(-j2\pi f\tau_d).$$

The system is then equivalent to an attenuation factor

$$E(0)=\exp(-\alpha z/2)$$

and a time delay

$$\tau_d=L/v_g,$$

so that

$$E(z,t) = \exp(-\alpha L/2)\,E(0,t-\tau_d). \quad (16)$$

In this approximation the pulse travels at the group velocity (14) v_g , its intensity is attenuated by the factor $\exp(-\alpha L/2)$, but its initial shape is not altered. By comparison, in an ideal (lossless and non-dissipated) medium, $\alpha = 0$ and $\beta(\nu) = 2\pi\nu/c$, so that $v_g=c$ the pulse envelope travels at the speed of light in the medium and its height and shape are not altered.

Dispersion Coefficient

Since the group velocity

$$v_{gr} = 2\pi/(d\beta/d\nu)$$

is itself frequency dependent, different frequency component of the pulse undergo different delays

$$\tau_d = L/v_{gr}.$$

As a result, the pulse spreads and its shape is altered.

Two identical pulses of central frequencies ν and $\nu + \delta\nu$ suffer a differential delay:

$$\delta\tau = \frac{d\tau_d}{d\nu}\delta\nu = \frac{d}{d\nu}(\frac{L}{v})\delta\nu = D_\nu L\delta\nu \quad (17)$$

If $D_\nu > 0$ (normal dispersion), the travel time for the higher – frequency component is longer than the travel time for the lower – frequency component. Thus shorter – wave length components are slower. Normal dispersion occurs in glass in the visible band.

If the pulse has a spectral width σ_ν (Hz) then:

$$\sigma_\tau = |D_\nu|\sigma_\nu L \quad (18)$$

is an estimate of the spread of its temporal width. The dispersion coefficient D_ν is therefore a measure of the pulse time broadening per unit spectral width per unit distance (s/m·Hz).

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