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ROBUST ESTIMATION OF STATIONARY PROCESS

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Abstract: *The use of microprocessor measurement system gives the opportunity to improve the precision of the measurements. It is possible by realization of algorithms, taking in to account the changing conditions of measurement experiment. In the paper it is suggested an adaptive algorithm for calculation of the mathematical expectation* M_X *in the conditions of insufficient knowledge about the interval of correlation* $\tau_C \in [\tau_{C\min}, \tau_{C\max}]$.

Keywords: *Robust estimation, random processes, precision of the measurement, stochastic measurements.*

1. INTRODUCTION

During the practice of stochastic measurements, the unknown correlation characteristics of the random processes of the correlation interval determine the optimal interval of digitization and the necessary sample quantity [1]. The real value of the dispersion of the statistical errors σ_{KB}^2 while defining the mathematical expectation M_X of stationary ergodicity accidental process *X(t)* is determined by the following [2]:

$$\sigma_{KB}^2 = \frac{\sigma_X^2}{N} \qquad (1)$$

where σ_X is the root-mean-square value of *X(t)*,

N - number of the uncorrelation digital value of accidental process.

Therefore, the interval of averaging T_A for respectively value of σ_{KB}, will be determine of equation [3]:

$$T_A = N.\Delta t_D = \frac{\sigma_X^2}{\sigma_{KB}^2}.\Delta t_D \qquad (2)$$

where Δt_D is an interval of digital.

2. PROBLEM FORMULATION

In values of the interval of averaging T_A less then desired one, aren't done the requirements for precision of the measurement, and when the values are bigger it is observed unjustify decrease of the response time of the measuring process.

That determine stohastical significance for the value of τ_K in positioning parameters on the algorithm.

3. PROBLEM SOLUTION

The algorithm is based on an iterative procedure that is characterized by use of averaging in every step upon sample size. The result from the averaging of *l*-th step is determined as a function of the current time t, by equation (3):

$$M_{X_j,l}(t) = \frac{1}{m_l}.\sum_{s=1}^{m_l} X_K^j(t_S) \qquad (3)$$

where $X_K^j(t)$ is *k*-th realization of the rendam process *X(t)*, conform to j-th measurement experiment,

$X_K^j(t_S)$ - member valuation of realization $X_K^j(t)$ at moment of time t_S,

$m_l = m_{l-1} + \Delta m_l$.

The procedure continues until execution of the condition (4):

$$\Delta M_{X_j,l} \leq C_{TR} \qquad (4)$$

where $\Delta M_{X_j,l} = M_{X_j,l} - M_{X_j,l-1}$

C_{TR} – level of the permissible error in valuation of the mathematical expectation of the process.

Let's define probability characteristics of the iteration measurement procedure, that is function in time *t* [1]:

- $V_l(t)$ – probability for execution of condition (3) till the *l*-th step;
- $P_l(t)$ – probability for execution of condition (3) during the *l*-th step;
- $P_{\Sigma l}(t)$ – probability for execution of condition (3) during the *l*-th step or preceding steps;

From equation (3) and (4), we receive:

$$\Delta M_{X_j,l}(t) = \frac{1}{m_l}.\sum_{m_{l-1}}^{m_l} X_K^j(t_S) - \\ - \frac{\Delta m_l}{m_{l-1}.m_l}.\sum_{s=1}^{m_{l-1}} X_K^j(t_S) \qquad (5)$$

For large Δm_l, the definite properties of the sums of the random quantities are valid. Because of that, the distribution of the probability $\Delta \tilde{M}_{X_j,l}$ is near to the normal one. In case of Gaussian probability distribution of *X(t)*, the distribution of the estimates $\Delta \tilde{M}_{X_j,l}$, for all magnitudes of Δm_l is normal, too. That is why in the next analysis it is accepted that the probability distribution of the values $\Delta M_{X_j,l}$ is normal.

In case of ungaussian initial random process *X(t)* and small Δm_l the law for distribution of the probability $\Delta M_{X_j,l}$, could be determined, by convolution of untrivial laws of distribution [4]. It's not difficult to prove that the mathematical expectation of $\Delta M_{X_j,l}$ equal to zero, but the dispersion is defined as follows:

$$D\left[\Delta M_{X_j,l}\right] = \frac{\Delta m_l}{m_l^2}.\sigma_X^2.\left(1 + \frac{\Delta m_l}{m_{l-1}}\right) = \sigma_l^2 \qquad (6)$$

where σ_X^2 is the dispersion of ungaussian random process *X(t)*.

Therefore, it can be defined the probability dension as it is shown in equation (7):

$$W\left[\Delta M_{X_j,l}\right] = \frac{1}{\sigma_l\sqrt{2\pi}}.\exp\left[\frac{\left(\Delta M_{X_j,l}\right)^2}{2\sigma_l^2}\right] \qquad (7)$$

Therefore, the probability $V_l(t)$ will be defined as it is shown in equation(8):

$$V_l(t) = \int_{-C_{TR}}^{C_{TR}} W\left(\Delta M_{X_j,l}\right).d\left(\Delta M_{X_j,l}\right) = \Phi\left(\frac{C_{TR}}{\sigma_b}\right) \qquad (8)$$

where $\Phi(x) = \frac{2}{\sqrt{2\pi}}\int_0^x \exp\left(-\frac{x^2}{2}\right).dx$ is the integral of the probability.

Respectively for the probability $P_l(t)$ we get:

$$P_l(t) = V_l(t).\prod_{r=2}^{l-1}\left(1 - V_r\right) = \\ \Phi\left(\frac{C_{TR}}{\sigma_l}\right).\prod_{r=2}^{l-1}\left[1 - \Phi\left(\frac{C_{TR}}{\sigma_r}\right)\right] \qquad (9)$$

Finally for the probability $P_{\Sigma l}(t)$ we get the equation:

$$P_{\Sigma l}(t) = \sum_{i=2}^{l} P_i(t) = \\ \sum_{i=2}^{l}\Phi\left(\frac{C_{TR}}{\sigma_i}\right).\prod_{r=2}^{l-1}\left[1 - \Phi\left(\frac{C_{TR}}{\sigma_r}\right)\right] \qquad (10)$$

The obtained equation permit to calculate the average number of steps l_{AV} of the iterative procedure and the characteristics of the errors of the result under fixed values of the level C_{TR}, the dispersion σ_X^2 and the parameter Δm_l .

Therefore, we get:

$$l_{AV} = \sum_{l=2}^{l_{\max}} l.P_l(t) \qquad (11)$$

$$P_l(t) = 1 - P_{\Sigma l_{\max}-1}$$

where Δt_D is an interval of digital.

Therefore we get for dispersion:

$$\sigma_{KB\,l}^2 = \sum_{l=2}^{l_{\max}} \sigma_{KB\,l}^2.P_l(t) \qquad (12)$$

where $\sigma^2_{KBl} = \dfrac{\sigma^2_X}{m_l}$.

Equation (12) permits to be determined the level C_{TR}, necessary for assurance the needed precision of the dispersion σ^2_{KB}. The proposed algorithm can be used not only for defining of mathematical expectation, but for defining of all other probability characteristics of the random process $X(t)$. In the general case for the probability characteristics θ of equation (3) will be got:

$$\theta_{X_j,l}(t) = \frac{1}{m_l} . \sum_{s=1}^{m_l} g\left[X_K^j(t_S)\right] \qquad (13)$$

where $g\left[X_K^j(t_S)\right]$ is transformation in the base of the given probability characteristics θ of the random process $X(t)$.

Therefore equation (4) will have the next general kind:

$$\Delta\theta_{X_j,l} = \theta_{X_j,l} - \theta_{X_j,l-1} \qquad (14)$$

The reasons for defining of characteristics of the errors are similary.

4. CONCLUSIONS

The proposed iterative procedure for equations (3) to (12) permits an increasing of the quality of the measurements in the conditions of small a priori knowledge for the properties of the random process and its probability characteristics.

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