



NONLINEAR PARAMETER IDENTIFICATION OF RISK TECHNICAL SYSTEMS

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Abstract: This paper deals with an optimal method concerning nonlinear parameter identification of risk technical systems (automobile and railway transport, aircrafts, marine and river transport, chemical installations, munitions, information society suffering by terrorism). Unknown states of the model are built by sliding observers which converge in a finite time. Due to this property, it is possible to derive equations of the model in order to obtain an estimation law which converges to the nominal values of the parameters also in the finite time.

Keywords: Nonlinear Parameter Identification; Risk Technical Systems

1. INTRODUCTION

While identification in linear context it is now well established and it is widely applied, methods for nonlinear identification of *risk technical systems* (RTS) are nowadays uncommon. This paper deals with a new method to identify parameters of some nonlinear models. In order to identify unknown parameters, we build observers which estimate unknown states. Our approach is inspired from methods based on sliding observers. This kind of observers was introduced by Slotine, Hedrick and Misawa [1] and then used by many authors in various control problems. Our idea here is to generalize to nonlinear parameter identification the work by Utkin [2] on linear parameter identification. For instance, we will use low-pass filters in order to approximate the equivalent control and in order to avoid the chattering problems. The main advantage of our approach is its robustness and the fact that it converges in a finite time.

After describing our observer in section 2, we will apply the proposed method in section 3 in order to design a new identifier for a general class of nonlinear parameter systems $\dot{x} = f(x, \theta)$ where $x \in R^n$ and $\theta \in R^p$ is the vector of unknown parameters.

2. DESIGN OF A SLIDING OBSERVER

Let us consider the following class of nonlinear systems:

$$\dot{x}_i = x_{i+1}, \quad i = 1, \dots, n-1, \quad (1)$$

$$\dot{x}_n = f(x_1, x_2, \dots, x_{n-1}, x_n), \quad (2)$$

$$y = x_1, \quad (3)$$

where $x = (x_1, x_2, \dots, x_n)^T \in R^n$ and with $f(0) = 0$.

We will assume that the RTS is stable around the equilibrium point $x_0 = 0$ and that the region of attraction of $x_0 = 0$ is contained in a known ball of radius ρ . In the light of the observer designed by Drakunov and Utkin [3] and then studied by Ahmed-Ali [4] and Ahmed-Ali and Lamnabhi-Lagarrigue [5], let us consider the following sliding observer:

$$\dot{\xi}_1 = k_1 \cdot \text{sign}(x_1 - \xi_1), \quad (4)$$

$$\dot{z}_1 = \frac{1}{\tau_1} [\text{sign}(x_1 - \xi_1) - z_1], \quad (5)$$

$$\dot{\xi}_i = k_i \cdot \text{sign}(k_{i-1} z_{i-1} - \xi_i), \quad i = 2, \dots, n-1, \quad (6)$$

$$\dot{z}_i = \frac{1}{\tau_i} [\text{sign}(k_{i-1} z_{i-1} - \xi_i) - z_i], \quad (7)$$

$$\dot{\xi}_n = f(x_1, k_1 z_1, \dots, k_{n-1} z_{n-1}) + k_n (k_{n-1} z_{n-1} - \xi_n), \quad (8)$$

where $k_i, i = 1, \dots, n$ and $\tau_i, i = 1, \dots, n-1$ are suitable positive constants which will be determined later on. By defining the observation errors $e_i = x_i - \xi_i$ for

$i=1, \dots, n$ and the new states $\mu_i = x_i - k_{n-1}z_{n-1}$, for $i=2, \dots, n$ we obtain equations:

$$\dot{e}_i = x_{i+1} - k_i \text{sign}(x_i - \xi_i - \mu_i), \quad i=1, \dots, n-1 \quad (9)$$

$$\dot{\mu}_i = x_{i+1} + \frac{x_i - \mu_i}{\tau_{i-1}} - \frac{k_{i-1}}{\tau_{i-1}} \text{sign}(x_{i-1} - \xi_{i-1} - \mu_{i-1}), \quad i=2, \dots, n-1, \quad (10)$$

$$\dot{e}_n = f(x) - f(x_1, x_2 - \mu_2, \dots, x_n - \mu_n) - k_n \text{sign}(x_n - \xi_n - \mu_n), \quad (11)$$

$$\dot{\mu}_n = f(x) + \frac{x_n - \mu_n}{\tau_{n-1}} - \frac{k_{n-1}}{\tau_{n-1}} \cdot \text{sign}(x_{n-1} - \xi_{n-1} - \mu_{n-1}) \quad (12)$$

Proposition:

For any initial condition $x_0 \in R^n$ in the region of attraction, we can find positive constants, k_1, k_2, \dots, k_n that are large enough and positive constants, $\tau_1, \tau_2, \dots, \tau_{n-1}$ that are small enough such that the observation errors e_1, e_2, \dots, e_{n-1} and the states μ_2, \dots, μ_n converge to zero in a finite time and such that observation error e_n converges asymptotically to zero.

Proof:

Step 1: Let us define the Lyapunov function $V_1 = 0,5 \cdot e_1^2$. Its time derivative is given by equation:

$$\begin{aligned} \dot{V}_1 &= e_1 \cdot \dot{e}_1 = e_1 [x_2 - k_1 \text{sign}(e_1)] = \\ &= e_1 x_2 - k_1 \cdot |e_1| \end{aligned} \quad (13)$$

therefore: $\dot{V}_1 \leq |e_1| \cdot |x_2|_{\max} - k_1 \cdot |e_1|$.

Hence, if we chose the constant k_1 such that $k_1 > |x_2|_{\max}$, \dot{V}_1 will be negative definite and a sliding mode is reached after a finite time denoted by t_1 , the sliding surface being e_1 . This allows us to write that equation $e_1(t) = \dot{e}_1 = 0, \forall t \geq t_1$. At this first step, we then conclude that we can find a time t_1 and a positive constant k_1 such that:

$$\forall t \geq t_1, \quad \begin{cases} \xi_1 = x_1, \\ x_2 = k_1 \text{sign}(e_1). \end{cases} \quad (14)$$

Step 2: Let us first consider the Lyapunov function $W_2 = 0,5 \cdot \mu_2^2$. Its time derivative is given by equation:

$$\begin{aligned} \dot{W}_2 &= \mu_2 \cdot \dot{\mu}_2 = \mu_2 \cdot \left[x_3 - \frac{k_1}{\tau_1} \cdot \right. \\ &\left. \cdot \text{sign}(x_1 - \xi_1) + \frac{x_2 - \mu_2}{\tau_1} \right]. \end{aligned} \quad (15)$$

Using (14) leads to, for all $t \geq t_1$, $\dot{W}_2 = \mu_2 \cdot (x_3 - \mu_2 / \tau_1)$. This implies that if we choose the positive constant τ_1 such that equation:

$$\dot{W}_2 \geq |\mu_2| \cdot |x_3|_{\max} - |\mu_2|^2 / \tau_1 < 0, \quad (16)$$

that is, if $\tau_1 \leq |\mu_2| / |x_3|_{\max}$.

Then there exists a finite time $t'_2 \geq t_1$ such that a sliding mode occurs on:

$$\mu_2 = 0. \quad (17)$$

Let us now consider the full state Lyapunov function:

$$V_2 = V_1 + 0,5e_2^2 + 0,5\mu_2^2 \quad (18)$$

It follows that:

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 + \mu_2 \dot{\mu}_2. \quad (19)$$

Now, by using (14) and (17), we obtain:

$$\dot{V}_2 \leq |e_2| \cdot |x_3|_{\max} - k_2 \cdot |e_2|. \quad (20)$$

Hence, if we choose the constant k_2 such that $k_2 > |x_3|_{\max}$, \dot{V}_2 will be negative definite and a sliding mode is reached after a finite time denoted t_2 , the sliding surface being $e_2 = 0$. This allows us to write that $e_2(t) = \dot{e}_2 = 0, \forall t \geq t_2$, at this second step, we then conclude that we can find a time $t_2 \geq t'_2 \geq t_1 \geq 0$ and positive constant τ_1 and k_2 such that:

$$\forall t \geq t_1 \quad \begin{cases} \xi_2 = x_2, \\ x_3 = k_2 \cdot \text{sign}(e_2). \end{cases} \quad (21)$$

Step n : This step is almost the same as the previous one. Let us first consider:

$$W_n = \frac{1}{2} \sum_{i=2}^{i=n} \mu_i^2. \quad (22)$$

It can easily be shown that we can find a time $t'_n \geq t_{n-1}$ such that a sliding mode occurs on equation:

$$\mu_n = 0, \quad (23)$$

provided that the positive constant τ_{n-1} is chosen such that $\tau_{n-1} \leq |\mu_n| / |f(x)|_{\max}$.

The end of the proof is shown by considering the full state Lyapunov function:

$$V_n = \frac{1}{2} \sum_{i=1}^{i=n} e_i^2 + \frac{1}{2} \sum_{i=2}^{i=n} \mu_i^2. \quad (24)$$

First, we note that, for all $t \geq t_{n-1}$:

$$\dot{V}_n = e_n \dot{e}_n + \mu_n \dot{\mu}_n, \quad (25)$$

and hence by using (23), it follows that, for all $t \geq t'_n \geq t_{n-1}$ is done $\dot{V}_n \leq -k_n \cdot e_n^2$. We then conclude that the observation error e_n converges asymptotical to zero.

Remark: The important property of our sliding observer is that it converges in finite time expert for the last observation error. The convergence in a finite time (see Slotine et Li [6]) can be quickly explained as follows. Let us define a time varying surface $S(t)$ of the form:

$$s(x, t) = \left(\frac{d}{dt} + \lambda \right)^{n-1} \cdot (x - x_t), \quad (26)$$

where x_t is the trajectory. The sliding condition is reminding a Lyapunov function $0.5 \cdot (ds^2/dt) \leq -\eta \cdot |s|$. This implies that $s \leq -\eta$ if S is positive. Let t_1 be the time required to hit the surface $s = 0$. By integrating the equation between $t = t_0 = 0$ and $t = t_1$ leads to $s(t_1) - s(t_0) \leq -\eta \cdot (t_1 - t_0)$. However, $s(t_1) = 0$ and $t_0 = 0$. Therefore we have $t_1 \leq s(t_0) / \eta$, which illustrates the convergence to the surface in a finite time.

3. DESIGN OF A SLIDING IDENTIFIER

Let us consider the general class of nonlinear systems $\dot{x} = f(x, \theta)$, where $x \in \mathbb{R}^n$ represents the vector of states and $\theta \in \mathbb{R}^p$ is the vector of unknown parameters. In the following we will assume that the system is identifiable and that the function f is sufficiently derivable. This means that there exist two functions Θ and Φ such that:

$$\theta = \Theta(x, \dot{x}, \dots, x^{(q)}), \quad (27)$$

$$x^{(q+1)} = \Phi(x, \dot{x}, \dots, x^{(q)}), \quad (28)$$

where q is a positive integer. Therefore, the identification problem can be stated as the search for an observer converging in finite time of the following system:

$$\begin{cases} \frac{d\xi_1}{dt} = \xi_2, \\ \frac{d\xi_2}{dt} = \xi_3, \\ \frac{d\xi_{q+1}}{dt} = F(\xi_1, \xi_2, \dots, \xi_q). \end{cases} \quad (29)$$

with the true vector of parameters θ obtained by the design of an adoption law, denoted in the following by $\hat{\theta}(t)$.

By applying the design of the sliding observer of the previous section to the system (29), we know that there exist large enough gains $k_i, i=1, \dots, n$ and small enough $\tau_i, i=1, \dots, n-1$, such the new states $\xi_i, i=1, \dots, q$ converge in a finite time to zero. We then immediately deduce that the adaptation law:

$$\hat{\theta}(t) = \Theta(\xi_1, k_1 \xi_1, \dots, k_{q-1} \xi_{q-1}), \quad (30)$$

converges also in finite time, to the nominal values of the unknown parameters.

In order to illustrate this result, we show how to find the nominal values for a synchronous machine using a quite simple model.

4. SIMULATION RESULTS

4.1. DESIGN OF THE MODEL

Let us consider a synchronous machine for aircrafts described by the following simple model of three state equations with three unknown parameters:

$$\begin{cases} \dot{i}_d = -\frac{R_S}{L}i_d + p\Omega i_q + \frac{1}{L}v_d, \\ \dot{i}_q = -\frac{R_S}{L}i_q - p\Omega i_d - p\frac{\phi_f}{L}\Omega + \frac{1}{L}v_q, \\ \dot{\Omega} = -\frac{f}{J}\Omega + \frac{p}{J}\phi_f i_q. \end{cases} \quad (31)$$

where i_d, i_q, v_d and v_q are respectively the stator currents and the stator voltages in the Park reference. Ω is the mechanical speed. All the three states i_d, i_q, Ω are assumed to be measurable and the friction parameter f , the inertia J , and the number of poles p , are assumed to be known. The unknown parameters that we wish to identify are the resistor R_S , the inductance L and the flux ϕ_f .

Finally, the reference values used in the simulation are given by:

$$\theta_1 = R_S/L = 9,5; \theta_2 = 1/L = 1,332; \theta_3 = \phi_f = 0,1 \quad (32)$$

4.2. IDENTIFIABLY

First of all, we need to know the identifiability of the RTS i.e., if we can identify the three parameters. Rewriting the RTS described by (31) using (32) the notation, $x_1 = i_d, x_2 = i_q, x_3 = \Omega$ gives:

$$\begin{cases} \dot{x}_1 = -\theta_1 x_1 + p x_3 x_2 + \theta_2 v_d, \\ \dot{x}_2 = -\theta_1 x_2 - p x_3 x_1 - p \theta_3 \theta_2 x_3 + \theta_2 v_q, \\ \dot{x}_3 = -\frac{f}{J} x_3 + \frac{p}{J} \theta_3 x_2. \end{cases} \quad (33)$$

In order to prove the identifiability, let us apply a method developed by Lecourtier, Lamnabhi-Lagarrigue and Walter [8] and based on generating power series. Let us consider the vector fields associated with equation (33),

$$F_0(x, \theta)[.] = (-\theta_1 x_1 + p x_3 x_2) \frac{\partial[.]}{\partial x_1} + \Xi(x, \theta), \quad (34)$$

where

$$\Xi(x, \theta) = (-\theta_1 x_2 - p x_3 x_1 - p \theta_3 \theta_2 x_3) \cdot \frac{\partial[.]}{\partial x_2} + \Psi(x, \theta),$$

$$\Psi(x, \theta) = \left(\frac{p}{J} \theta_3 x_2 - \frac{f}{J} x_3 \right) \frac{\partial[.]}{\partial x_3}.$$

Is analogy consider the vector fields with equation:

$$F_1(x, \theta)[.] = \theta_2 \frac{\partial[.]}{\partial x_1}, \quad (35)$$

$$F_2(x, \theta)[.] = \theta_2 \frac{\partial[.]}{\partial x_2}. \quad (36)$$

Let us now compute the following Lie derivatives $F_1(F_0(x, \theta)[.])$, $F_2(F_0(x, \theta)[.])$ and $F_1(F_0(F_0(x, \theta)[.]))$ applied to the arbitrary chosen output $y = x_1 + x_3$. That leads to equations:

$$F_1(F_0(x, \theta)[y]) = -\theta_1 \theta_2, \quad (37)$$

$$F_2(F_0(x, \theta)[y]) = \frac{p}{J} \theta_3 \theta_2, \quad (38)$$

$$F_1(F_0(F_0(x, \theta)[y])) = \theta_2 \theta_1^2. \quad (39)$$

The correct identification of the RTS, is proven by showing that the nonlinear algebraic system, where $\theta' = (\theta'_1, \theta'_2, \theta'_3)$ is another triplet of parameters, has a unique solution $\theta = \theta'$. It this following the system equations:

$$\begin{cases} \theta_1 \theta_2 = \theta'_1 \theta'_2, \\ \theta_3 \theta_2 = \theta'_3 \theta'_2, \\ \theta_2 \theta_1^2 = \theta'_2 (\theta'_1)^2. \end{cases} \quad (40)$$

5. CONCLUSIONS

The conclusions obtained are summarized as follows:

1. In this paper, a new approach for the identification of parameters of risk technical systems was investigated.
2. This approach could be applied to a large class of nonlinear systems.
3. Moreover, this approach is robust towards noise compared to the classical method based on the recursive least square algorithm.

6. REFERENCES

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