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OPTIMAL DISIGN OF ENERGY SUPPLY NETS ON GRAPHS

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Abstract: *In the design and operation of energy intensive systems, the possibility of improving the system's efficiency is very important to explore. The main way of improving efficiency is through optimisation. This paper describes the application of exergy topological models and, in particular, the graph of thermoeconomical expenditure for thermoeconomical optimal design s of circled nets for energy supply (CNES). The questions of thermoeconomical optimisation of CNES, as well as suggested modelling algorithms, are illustrated in the numerical example of the optimisation of a energy supply system for a city with seven regions of energy consumption.*

Keywords: *Optimal design, energy supply nets, thermoeconomic.*

1. INTRODUCTION

The processes taking place in complex energy intensive systems are characterized by the mutual transformation of quantitatively different power resources. The thermo-economical optimisation of CNES is based on thermodynamic analysis, which requires the combined application of both laws of thermodynamics and demands the exergy approach ([1], [2]).

Exergetic methods are universal and make it possible to estimate the fluxes and balances of energy for every element of the system using a common criterion of efficiency.

Therefore, the exergetic methods are meaningful in analysis and calculations.

Meanwhile, the increasing complexity of optimisation problems requires more effective and powerful mathematical methods. Therefore, during the last few years, many papers with different applications of exergetic methods and the thermo-economical approach have been published (see for example [3], [4], [5], [7]).

The above referenced papers, as well as the author's past investigations [7-12], show that one of the most effective mathematical methods used for exergetic analysis and thermoeconomical optimisation involve graph theory [13]. The usefulness of graph models can also be demonstrated by their flexibility and wide range of possible applications.

The exergy topological method includes the sole use or combination of exergy flow graphs [7-9] and thermoeconomical graphs [10-12].This paper describes the application of exergy topological models and, in particular, the graph of thermo-economical expenditure for thermoeconomical optimisation of CNES.

2. METHOD AND ALGORITHM OF OPTIMAL SYNTHESIS OF CNES

Let's assume that the CNES contains **m** customers and the possible methods of connection of these customers by a net are known.

Then, for this CNES, in accordance with rules given in [11,12], the graph of thermoeconomical expenditure can be built. Shown in Fig. 1 is a graph whose nodes multitude $A=\{a_1, a_2, ..., a_i, ..., a_m\}$ corresponds to the customers and arcs multitude $U=\{a_i, a_j\}$; $i \neq j$; $i = 1, 2, ..., m$; $j = 1,2, ..., m$; to the appropriate parts of CNES between nodes a_i, a_j. Each arc U_{ij} has thermoeconomical expenditure Z_{ij} as it is shown in the matrix of thermoeconomical expenditure (see Fig. 2.) and the graph in Fig. 1.

Then, by minimizing the sum shown in Eq. (1), the problem of optimal thermoeconomical synthesis can be solved.

$$Z_{\Sigma}^{\min} = \min \sum_i \sum_j Z_{ij} \quad (1)$$

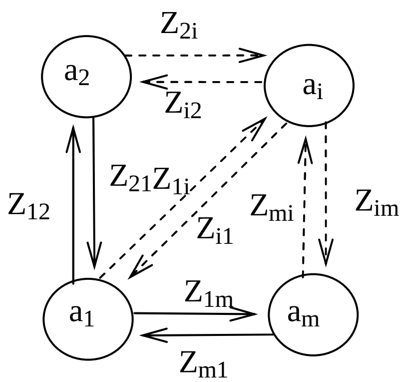

Fig.1 – Graph of thermoeconomical expenditure.

	a_1	a_2	...	a_j	...	a_m
a_1	Z_{11}	Z_{12}		Z_{1j}		Z_{1m}
a_2	Z_{21}	Z_{22}		Z_{2j}		Z_{2m}
...						
a_i	Z_{i1}	Z_{i2}		Z_{ij}		Z_{im}
...						
a_m	Z_{m1}	Z_{m2}	…	Z_{mj}	…	Z_{mm}

Fig.2 – Matrix of thermoeconomical expenditure corresponding to a graph in Fig.1.

Given below is the matrix form of a special algorithm for the optimal synthesis of CNES based on the finding of a Hamilton contour [13] in the graph of thermoeconomical expenditure

$Z_U = (A, U)$.

The algorithm consists of following main steps:

Step 1. Calculate the possible thermoeconomical expenditure $Z_{ij} = Z(a_i, a_j)$, $\forall a_i \in A$,

$\forall a_j \in A$ and form a square matrix of size **m x m** for the thermoeconomical expenditure (See Fig.2.).

Step 2. Find a minimum element in each i-th line of the matrix $Z_i^{min} = \min\{Z_{ij}\}$, j = 1,2..., m; i = 1,2..., m and subtract the element from all elements in this line.

Step 3. Check: are there any matrix columns that do not include zero elements?

If yes, then go to step 4.

If not, then each line and each column contain at least one zero member. Proceed to step 5.

Step 4. Find, in each j- column, that does not include the zero elements, a minimum element. This element will be $Z_j^{min} = \min\{Z_{ij}\}$, i = 1,2..., m; j = 1,2..., m. Now subtract Z_j^{min} from all elements of this column. The result will be an inclusive matrix yielding one zero element in each column and each line.

Step 5. Calculate the sum: $Z_\Sigma^0 = \sum_i Z_i^{min} + \sum_j Z_j^{min}$

This sum, Z_Σ^0, is the lower boundary of a set of the solutions and can be accepted as the root tree for the thermoeconomical expenditure.

It is understandable that if step 4 was not executed, then $Z_j^{min} = 0$.

Step 6. Select an arc, (a_k, a_l), for which

$$R^{max}(a_k, a_l) = \max\{R(a_i, a_j)\}$$

$R(a_i, a_j)$ – the sum of the least element of i-th line and j-th column of a matrix. Zero element is located is the interception of these i-th line and j-th column.

Step 7. Find, in the tree of thermo-economical expenditure, a dangling vertex with the least boundary.

Step 8. Form the new vertex with a boundary equal the sum of the boundary of vertex in step 7 with value $R^{max}(a_k, a_l)$.

An adequate contour for this vertex will not use an arc (a_k, a_l).

Let's designate this property through $\overline{S}_{kl}$.

Step 9. Eliminate the k-th line and l-th column in the matrix corresponding to an element $R^{max}(a_k, a_l)$. Then the size of the matrix will decrease by a unit.

Step 10. Exchange a symbol, ∞, for the thermoeconomical expenditure of arcs, which permits finding contours of length smaller than the **m**-size.

Step 11. Check: Is the size of the matrix obtained in step 10 more than that of a unit?

If yes, then go to a step 12.

If not, then go to a step 19.

Steps 12, 13, 14, and 15 repeat steps 2, 3, 4, and 5, but these calculations are done with the matrix obtained in step 10 (instead of the initial matrix used in previously).

Step 16. Add the sum obtained in step 15 to the value for the boundary of vertex from which one splitting was done (in the first step, this is the boundary for a root tree of thermoeconomical expenditure).

The final result will be the boundary for the new dangling vertex - a contour will use an arc (a_k, a_l) that is adequate for the condition in step 6.

Step 17. Find the dangling vertex with the least boundary. If there are only a few dangling vertices with the same boundaries, then select a vertex that is characterized by property $S_{kl.}$ This step is essential in order to find arcs that are included in a Hamilton contour.

Step 18. Check: is the vertex found in step 17 built by applying the property $\overline{S}_{kl}$?

If yes, then go to a step 6.

If is not, then go to a step 9.

Step 19. The problem is solved - optimal pairs of elements (customers) a_i and a_j are found that correspond to the appropriate vertex sequence. By starting from the root tree and finishing with the dangling vertex for a matrix of unit size, the single contour CNES, with the minimum thermoeconomical expenditure in accordance with Eq. 1, can be determined.

3. NUMERICAL EXAMPLE OF OPTIMAL SYNTHESIS OF SINGLE CONTOUR CNES

Let's consider a problem of an optimal synthesis, single contour CNES for a city with seven regions of energy consumption (see scheme in Fig. 3.).

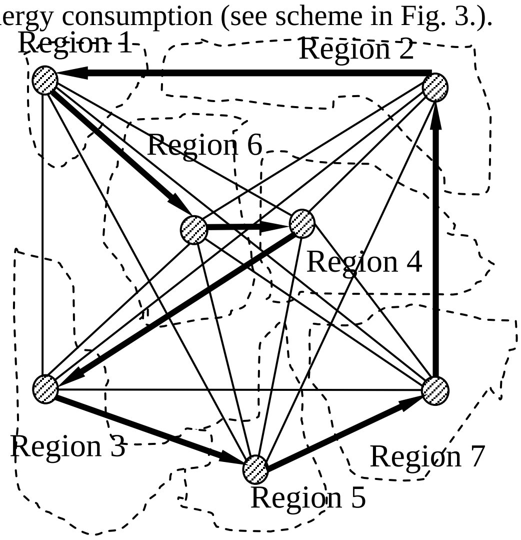

Fig.3 – Scheme CNES with seven region of energy consumption

The graph of thermoeconomical expenditure is given in Fig. 4, and the matrix of thermoeconomical expenditure, $\mathbf{M_1}$, is found in Fig. 5.

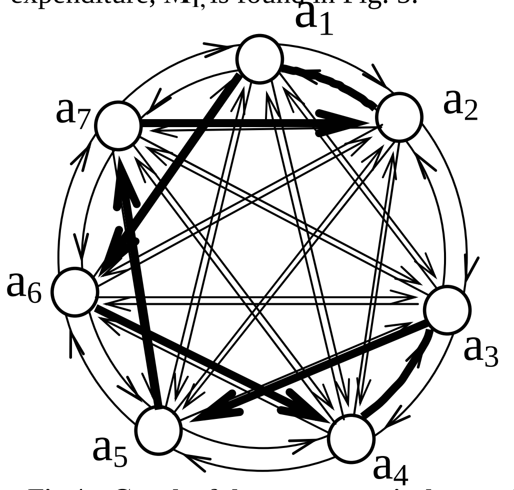

Fig.4 – Graph of thermoeconomical expenditure for scheme in Fig.3.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	∞	1050	1850	1200	6300	1100	1900
a_2	1050	∞	1700	1600	900	2500	400
a_3	1850	1700	∞	200	1200	1550	1500
a_4	1200	1600	200	∞	400	800	900
a_5	6300	900	1200	400	∞	300	450
a_6	1100	2500	1550	800	300	∞	950
a_7	1900	400	1500	900	450	950	∞

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	∞	0	800	150	5250	50	850
a_2	750	∞	1300	1200	500	2100	0
a_3	1650	1500	∞	0	100	1350	1300
a_4	100	1400	0	∞	200	600	700
a_5	6000	600	900	100	∞	0	150
a_6	800	2200	1250	500	0	∞	650
a_7	1500	0	1300	500	50	550	∞

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	∞	0	800	150	5250	50	850
a_2	650	∞	1300	1200	500	2100	0
a_3	1550	1500	∞	0	100	1350	1300
a_4	100	1400	0	∞	200	600	700
a_5	5900	600	900	100	∞	0	150
a_6	700	2200	1250	500	0	∞	650
a_7	1400	0	1300	500	50	550	∞

Fig.5 – Matrixes of thermoeconomical expenditures $\mathbf{M_1}$-$\mathbf{M_3}$.

The matrices of solution, $\mathbf{M_1}$-$\mathbf{M_{23}}$ (see Fig. 5-Fig. 9), as well as the tree of thermoeconomical expenditure (see Fig. 10), are obtained as a result of applying the suggested algorithm.

It is easy to see that the optimal single contour CNES (in Fig. 4 and Fig. 5 - designated by bold lines) contains the appropriate sequence of nodes (customers): (a_1, a_6, a_4, a_3, a_5, a_7, a_2, a_1).

The minimum thermoeconomical expenditure for this optimized CNES is 4400.

	a_1	a_2	a_4	a_5	a_6	a_7
a_1	∞	0	150	5250	50	850
a_2	650	∞	1200	500	2100	0
a_3	1550	1500	∞	100	1350	1300
a_5	5900	600	100	∞	0	150
a_6	700	2200	500	0	∞	650
a_7	1400	0	500	50	550	∞

	a_1	a_2	a_4	a_5	a_6	a_7
a_1	**∞**	**0**	**150**	**5250**	**50**	**850**
a_2	**650**	**∞**	**1200**	**500**	**2100**	**0**
a_3	**1450**	**1400**	**∞**	**0**	**1250**	**1200**
a_5	**5900**	**600**	**100**	**∞**	**0**	**150**
a_6	**700**	**2200**	**500**	**0**	**∞**	**650**
a_7	**1400**	**0**	**500**	**50**	**550**	**∞**

	a_1	a_2	a_4	a_5	a_6	a_7
a_1	**∞**	**0**	**50**	**5250**	**50**	**850**
a_2	**0**	**∞**	**1100**	**500**	**2100**	**0**
a_3	**800**	**1400**	**∞**	**0**	**1250**	**1200**
a_5	**5250**	**600**	**0**	**∞**	**0**	**150**
a_6	**50**	**2200**	**400**	**0**	**∞**	**650**
a_7	**750**	**0**	**400**	**50**	**550**	**∞**

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	**∞**	**0**	**800**	**150**	**5250**	**50**	**850**
a_2	**650**	**∞**	**1300**	**1200**	**500**	**2100**	**0**
a_4	**1550**	**1500**	**∞**	**0**	**100**	**1350**	**1300**
a_5	**0**	**1400**	**∞**	**∞**	**200**	**600**	**700**
a_6	**5900**	**600**	**900**	**100**	**∞**	**0**	**150**
a_7	**700**	**2200**	**1250**	**500**	**0**	**∞**	**650**
	1400	**0**	**1300**	**500**	**50**	**550**	**∞**

Fig.6 – Matrixes of thermoeconomical expenditures M_4-M_7

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	**∞**	**0**	**0**	**150**	**5250**	**50**	**850**
a_2	**650**	**∞**	**500**	**1200**	**500**	**2100**	**0**
a_3	**1550**	**1500**	**∞**	**0**	**100**	**1350**	**1300**
a_4	**0**	**1400**	**∞**	**∞**	**200**	**600**	**700**
a_5	**5900**	**600**	**100**	**100**	**∞**	**0**	**150**
a_6	**700**	**2200**	**450**	**500**	**0**	**∞**	**650**
a_7	**1400**	**0**	**500**	**500**	**50**	**550**	**∞**

	a_2	a_3	a_4	a_5	a_6	a_7
a_1	**0**	**0**	**∝**	**5250**	**50**	**850**
a_2	**∞**	**500**	**1200**	**500**	**2100**	**0**
a_3	**1500**	**∞**	**0**	**100**	**1350**	**1300**
a_5	**600**	**100**	**100**	**∞**	**0**	**150**
a_6	**2200**	**450**	**500**	**0**	**∞**	**650**
a_7	**0**	**500**	**500**	**50**	**550**	**∞**

	a_2	a_3	a_4	a_5	a_6
a_1	**0**	**0**	**∝**	**5250**	**50**
a_3	**1500**	**∞**	**200**	**100**	**1350**
a_5	**600**	**100**	**100**	**∞**	**0**
a_6	**2200**	**450**	**500**	**0**	**∞**
a_7	**∞**	**500**	**500**	**500**	**550**

	a_2	a_3	a_4	a_5	a_6
a_1	**0**	**0**	**∞**	**5250**	**50**
a_3	**1400**	**∞**	**0**	**0**	**1250**
a_5	**600**	**100**	**0**	**∞**	**0**
a_6	**2200**	**450**	**400**	**0**	**∞**
a_7	**∞**	**450**	**350**	**0**	**500**

Fig.7 – Matrixes of thermoeconomical expenditures M_8-M_{11}

	a_1	a_2	a_4	a_6	a_7
a_1	**∞**	**0**	**50**	**50**	**850**
a_2	**0**	**∞**	**1100**	**2100**	**0**
a_5	**5250**	**600**	**0**	**0**	**150**
a_6	**50**	**2200**	**400**	**∞**	**650**
a_7	**750**	**0**	**400**	**550**	**∞**

	a_1	a_2	a_4	a_6	a_7
a_1	**∞**	**0**	**50**	**50**	**850**
a2	**0**	**∞**	**1100**	**2100**	**0**
a_5	**5250**	**600**	**0**	**0**	**150**
a_6	**0**	**2150**	**350**	**∞**	**600**
a_7	**750**	**0**	**400**	**550**	**∞**

	a_1	a_4	a_6	a_7
a_1	**∞**	**50**	**50**	**850**
a_2	**0**	**1100**	**2100**	**∝**
a_5	**5250**	**0**	**0**	**150**
a_6	**0**	**350**	**∞**	**600**

	a_1	a_4	a_6	a_7
a_1	**∞**	**0**	**0**	**850**
a_2	**0**	**1100**	**2100**	**∞**
a_5	**5250**	**0**	**0**	**0**
a_6	**0**	**350**	**∞**	**450**

	a_3	a_4	a_5	a_6
a_3	**∞**	**0**	**0**	**1250**
a_5	**100**	**0**	**∞**	**0**
a_6	**450**	**400**	**0**	**∞**
a_7	**450**	**350**	**0**	**500**

	a_3	a_4	a_5	a_6
a_3	**∞**	**0**	**0**	**1250**
a_5	**0**	**0**	**∞**	**0**
a_6	**350**	**400**	**0**	**∞**
a_7	**350**	**350**	**0**	**500**

Fig.8 – Matrixes of thermoeconomical expenditures M_{12}-M_1.

	a_3	a_4	a_5
a_3	∞	**0**	**0**
a_6	**350**	**400**	**0**
a_7	**350**	**350**	**0**

	a_3	a_4	a_5
a_3	∞	**0**	**0**
a_6	**0**	**400**	**0**
a_7	**0**	**350**	**0**

	a_4	a_6	a_7
a_1	**0**	**0**	**850**
a_5	**0**	**0**	**0**
a_6	**350**	∞	**450**

	a_4	a_6	a_7
a_1	**0**	**0**	**850**
a_5	**0**	**0**	**0**
a_6	**0**	∞	**100**

	a_4	a_6
a_1	**0**	**0**
a_6	**0**	∞

	a_4
a_6	**0**

Fig.9 – Matrixes of thermoeconomical expenditures M_{18}-M_{23}.

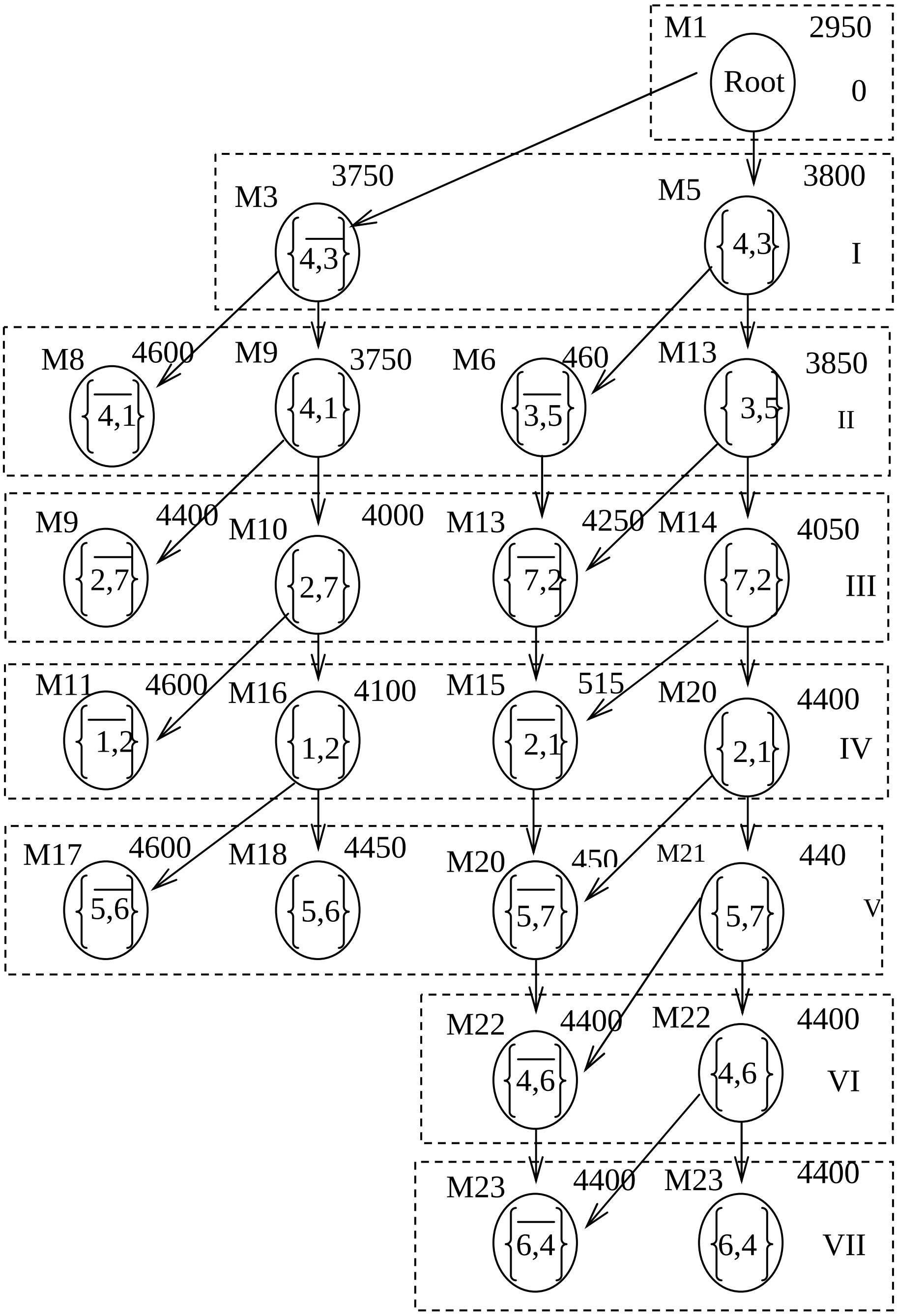

Fig.10 – Tree of thermoeconomical expenditure.

4. CONCLUSION

The problem of optimisation for CNES has to be solved separately from the problem of optimisation of other energy intensive systems. On the basis of the unique features of such a system, it is possible to build an effective procedure for optimisation.

In this paper, the authors develop and analyse a graph of thermoeconomical expenditure through a unique approach. The suggested method and algorithm are based on seeking a Hamilton contour in the graph of thermoeconomical expenditure.

The method is illustrated with a numerical example of a single contour CNES of a city with seven regions of energy consumption.

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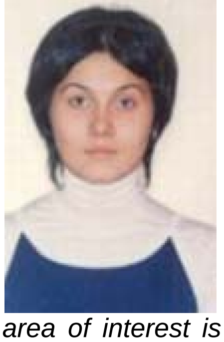

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