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The Improved Method for Identifying Parameters of Interval Nonlinear Models of Static Systems

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ABSTRACT The article discusses the method of identifying parameters for interval nonlinear models of static systems. The method is based on solving an optimization problem with a smooth objective function. Additional coefficients are added to the objective function's variables to solve the optimization problem, complicating the computational procedures. The computational complexity of quasi-Newton methods used to solve the optimization problem is analyzed. Excessive computational complexity is caused by many iterations when transforming the value of the objective function to zero. To address this, the article proposes using the optimization stop criterion based on the determination of the model's adequacy at the current iteration of the computational optimization procedure. Numerical experiments were conducted to identify nonlinear models of depending the pH of the environment in the fermenter of the biogas plant on influencing factors. It was established that the proposed criterion reduced the number of iterations by 4.5 times, which is proportional to the same reduction in the number of calculations of the objective function. Gotten results are also important for reducing the computational complexity of algorithms of structural identification of these models.

KEYWORDS interval model; static systems; parameter identification; objective function; optimization stop criterion; computational complexity.

I. INTRODUCTION

THE problem of parametric identification of mathematical models of static systems is formulated as an optimization problem [1]. As is known [2], the complexity of this problem depends on the form of the equation that describes the model (whether linear or nonlinear), the number of model parameters, and also on the way of representing the inaccuracy in the experimental data. If the measurement errors of the devices are taken into account in the data, then in this case the results of observations are presented in interval form [3, 4]. In this case, the obtained interval model of the static system has "guaranteed" prognostic properties, but the computational scheme for identifying the parameters of such a model is quite complex [5]. This is especially observed in the case of nonlinearity of the algebraic equation that describes the mathematical model [1, 6].

Currently, the identification methods of linear interval models of static systems, which are based on computing schemes of linear programming, have been sufficiently developed [7]. In cases where the equation that describes the mathematical model is nonlinear in the procedures for identifying model parameters under the conditions of interval data analysis, metaheuristic algorithms are used, in particular, artificial bee colony algorithms [8-10], since optimization problems, in this case, are complex with nonlinear multi extremal discrete objective functions [11].

In recent years, instead of these problems, a problem with smooth objective functions has been formulated and solved. In this case, the objective function minimizes the squared error between the values selected on the intervals of the experimental data and the values predicted on the same intervals by one model, which is selected from the corridor of interval models. So, this approach makes it possible to use a smooth objective function in the problem of parametric identification, which is an optimization problem [1]. At the same time, this leads to an increase in the dimension of the optimization problem by N

unknown parameters of the solution, which corresponds to the number of constraints on the objective function [1]. Under these conditions, there is a need to find additional conditions for the completion of the optimization problem of parameter identification to avoid an increase in its computational complexity due to dimensionality. This task is the subject of research in this article.

II. MATERIAL AND METHODS

A. STATEMENT OF THE TASK

In many cases, the relationships between the observed properties of complex objects and factors reflecting the influence of the external environment on them are presented in the form of nonlinear algebraic equations. Static systems, in such cases, describe the functional dependencies between the input values of the factors affecting the system and the output values of the characteristics in the form of an expression [2]:

$$y(\vec{\beta},\vec{X}) = f_1(\vec{\beta},\vec{X})+\ldots+f_m(\vec{\beta},\vec{X}), \tag{1}$$

where $y(\vec{\beta},\vec{X})$ is the simulated value of the characteristic of the static system; $\vec{\beta}$ is a vector of model parameters (parameters in the model have no physical meaning and are unknown coefficients); $f_1(\vec{\beta},\vec{X})+\ldots+f_m(\vec{\beta},\vec{X})$ is a set of basis functions both from the vector of input variables $\vec{X}$ and the vector of parameters $\vec{\beta}$ of the model.

The results of the experiment, which are necessary to identify the parameters of the nonlinear (in general form) model (2), are presented in the following form [1]:

$$\vec{X}_i \rightarrow [y_i^-; y_i^+],\ i = 1,\ldots,N, \tag{2}$$

where $[y_i^-; y_i^+]$ is the lower and upper limit of the interval values of the system characteristics for the given i-th measurement conditions, which are determined by the vector $\vec{X}_i$, for each of the $i = 1, \ldots, N$ measurements. In this case, the task of identifying the model in the form of expression (1) is to calculate the estimates of $\hat{\vec{\beta}}$ parameters. In the case when these estimates are calculated the mathematical model takes the following form:

$$\hat{y}(\hat{\vec{\beta}},\vec{X}) = f_1(\hat{\vec{\beta}},\vec{X})+\ldots+f_m(\hat{\vec{\beta}},\vec{X}), \tag{3}$$

where $\hat{y}(\hat{\vec{\beta}},\vec{X})$ is the simulated value of the system characteristic.

Based on the condition that the simulated values of the characteristics of the static system should belong to numerical intervals obtained experimentally [1]:

$$\hat{y}_i(\hat{\vec{\beta}},\vec{X}) \in [y_i^-; y_i^+],\ i = 1,..,N, \tag{4}$$

we get a mathematical problem for calculating estimates $\hat{\vec{\beta}}$ of the parameters vector $\vec{\beta}$ [1]:

$$y_i^- \le f_1\left(\hat{\vec{\beta}},\vec{X}_i\right) + \cdots + f_m\left(\hat{\vec{\beta}},\vec{X}_i\right) \le y_i^+, \quad i = 1, \ldots, N. \tag{5}$$

The resulting system (5) is an interval system of nonlinear algebraic equations (ISNAE) for unknown interval estimates of the parameter vector $\left[\vec{\beta}\right]$ [1]. The set of ISNAE solutions Ω determines the vector of estimates of model parameters. Taking into account the high (combinatorial) computational complexity of solving this ISNAE, in practice, only point estimates of parameters $\hat{\vec{\beta}}$. are calculated. In this case, to estimate the parameters, the optimization problem of the following form is solved [1]:

$$\delta\left(\hat{\vec{\beta}}\right) \xrightarrow{\hat{\vec{\beta}},\hat{\alpha}_i} min, \tag{6}$$

$$\hat{\vec{\beta}} \in [\hat{\vec{\beta}}^{low}; \hat{\vec{\beta}}^{up}], \tag{7}$$

$$\hat{\alpha}_i \in [0,1], i = 1, \ldots, N, \tag{8}$$

where $\hat{\alpha}_i$ are coefficients of linear combinations that define the points within the limits of the experimental data$[y_i^-; y_i^+]$.

In expression (6), the objective function $\delta(\hat{\vec{\beta}})$ is formed based on taking into account the restrictions established by the interval system of nonlinear algebraic equations (5). This objective function is a criterion for minimizing the quadratic error [1]:

$$\delta\left(\hat{\vec{\beta}}\right) = \sum_{i=1}^{N}\left(\hat{y}_i(\hat{\vec{\beta}},\vec{X}_i) - P([y_i^-; y_i^+], \alpha_i\right)^2 = = \sum_{i=1}^{N}\begin{pmatrix} f_1\left(\hat{\vec{\beta}},\vec{X}_i\right) + \cdots + f_m\left(\hat{\vec{\beta}},\vec{X}_i\right) - \\ (\alpha_i \cdot y_i^- + (1-\alpha_i)\cdot y_i^+) \end{pmatrix}^2. \tag{9}$$

In cases where the nonlinearity of the parameters complicates the objective function, stochastic optimization methods [12, 13], evolutionary [14-16] and metaheuristic algorithms, [17-19] are used. In particular, we can single out a method based on the application of the swarm intelligence algorithm of a colony of honey bees [11].

It is also worth noting that in the general case, the solution of the optimization problem (6)-(8) ensures the transformation of the value of the objective function to the zero value, i.e $\delta(\hat{\vec{\beta}}) = 0$. In this case, such a condition, taking into account the expansion of the parameter space of the solution vector by N, leads to a significant increase in the computational complexity of the parametric identification problem and requires additional research.

B. ADDITIONAL STOP-CRITERION FOR THE CALCULATION PROCEDURE OF OPTIMIZATION IN THE PROBLEM OF IDENTIFYING THE PARAMETERS OF INTERVAL MODELS

The MATLAB software libraries for optimization contain several quasi-Newton algorithms for solving both unconstrained and constrained nonlinear optimization problems [20-22]. In particular, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm belongs to quasi-Newton methods and is an iterative method for solving nonlinear optimization problems without constraints. BFGS determines the descent direction by pre-estimating the curvature based on the gradient. The estimation is performed by gradually improving the approximation to the Hessian matrix of the loss function obtained only from the gradient estimates (or approximate gradient estimates) using the generalized secant method.

The complexity of each iteration for the BFGS method is $O(m^2)$ with the addition of a component for the cost of calculating the value of the function and the gradient [23, 24]. This algorithm is a component of the interior point method, which generally determines its asymptotic computational complexity for the problem of parameter identification, which is described by the following dependence:

$$O(k, m, N) = O(k \cdot (N + m) \cdot m^2), \quad (10)$$

where k is the number of iterations during optimization.

The peculiarity of the problem (6) is that the objective function is built based on the convolution of constraints in the form of ISNAE (5). This feature of the problem statement increases the efficiency of the application of nonlinear optimization methods, as it eliminates nonlinear restrictions (there are restrictions on the values of the coefficients $\alpha_i, i = 1, \dots, N$), due to the addition of the parameter space with the coefficients α_i).

In this case, for the above method, we get the asymptotic computational complexity of the algorithm:

$$O(k, m, N) = O(k \cdot (m + N)^2). \quad (11)$$

As a rule, the number of executed iterations is determined by the stopping criteria, which are checked at the end of each iteration. For the interior point method in MATLAB R2023b Update 6, it is possible to limit the number of iterations (MaxIterations) and the number of target function calculations (MaxFunctionEvaluations), which does not guarantee obtaining the optimal value [25, 26]. Also, optimality tolerances (OptimalityTolerance), objective function growth (FunctionTolerance), and step size (StepTolerance) are used as optimization stopping criteria. The graphic interpretation of these criteria is shown in Fig. 1.

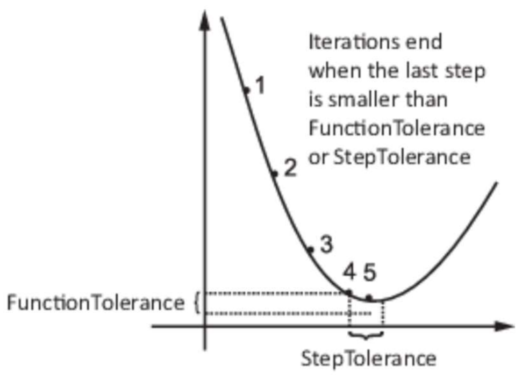


Figure 1. Criteria for terminating iterations in the interior point method [26].

In reference [1] it's stated that the minimization of the objective function to zero guarantees the optimality of the solution, that is, the adequacy of the model (6). Based on the basic assumptions on which the interval data analysis method is based, a set (corridor) of interval point nonlinear models is built based on the results of identifying model parameters, each of which adequately reflects the properties of a static system [3]. Accordingly, it can be assumed, that the solution to the problem (7), which is close to the optimal one, will ensure the construction of an adequate model, and conditions (4) will be satisfied for this model.

It is also worth noting that when the objective function is close to the minimum in the optimization procedure of the methods mentioned above, the number of iterations increases significantly. Accordingly, expression (4) can be used as a criterion for stopping the optimization procedure, which will allow to reduce the number of iterations.

Taking into account the above considerations, we obtain the asymptotic computational complexity of the method of identifying the parameters of nonlinear interval models of static systems in the following form:

$$O(k, m, N) = O(k \cdot ((N + m)^2 + N)). \quad (12)$$

The resulting expression (11) is due to the need to check condition (4) based on data of dimension N at each iteration.

In Fig. 2 shows graphs of the dependence of the computational complexity of the parameter identification method on the dimension of the optimization problem.

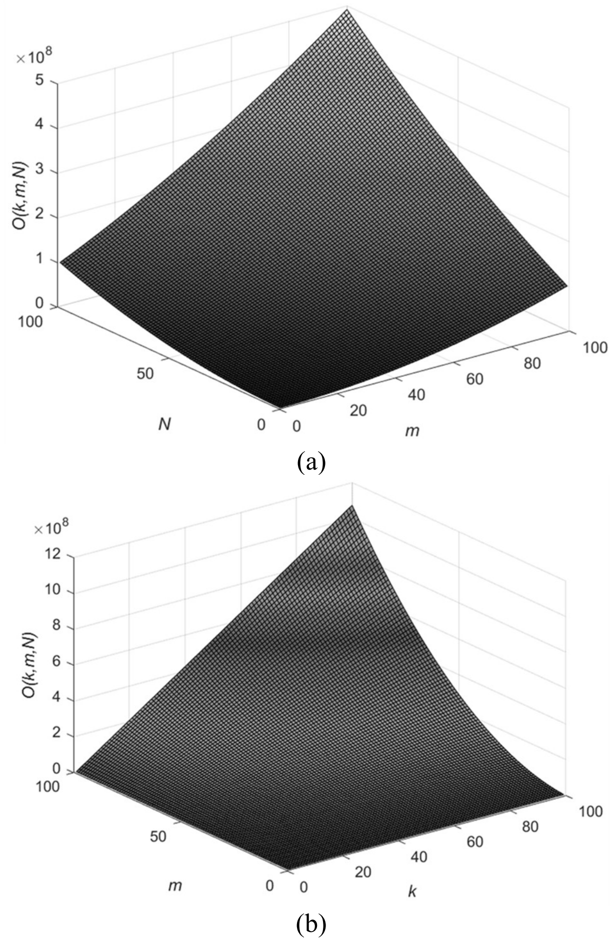


Figure 2. Graphs of the dependence of the computational complexity of the parameter identification method on the dimension of the optimization problem (7)

Fig. 2(a) shows the quadratic polynomial computational complexity based on the number of measurement data of the initial characteristics of the static system N and the number of model parameters m, which is explained by the inclusion of a linear combination coefficient for each measurement (observation) in the optimization parameter space. At the same time, when the number of iterations k increases, the computational complexity increases linearly (Fig. 2(b)). Therefore, one of the ways to reduce the computational complexity of the method is to reduce the number of iterations due to the use of the criterion for stopping the optimization of interval model parameters in the form of conditions (4).

III. RESULTS AND DISCUSSION

To investigate the effectiveness of using the proposed stop criterion for optimizing the parameters of interval nonlinear models, a series of computational experiments was conducted using procedures for identifying nonlinear interval models. In the course of numerical experiments, there were studies of the change in the number of iterations and, accordingly, the number of calculations of the value of the objective function $\delta(\vec{\beta})$.

Let us consider the effectiveness of the method based on the use of the proposed stopping criterion in the example of identifying a model that describes the pH dependence of the environment in the fermenter of a biogas plant [27]:

$$y(\vec{\beta},\vec{X}) = \beta_0 + \left(\frac{\beta_1 \cdot x_2}{1+\beta_2 \cdot x_1 + \beta_3 \cdot x_3 + \beta_4 \cdot x_4}\right)^{\beta_5} + \beta_6 \cdot {x_5}^{\beta_7} + \beta_8 \cdot {x_6}^{\beta_9}, \quad (13)$$

where x_1 – is the volume in м^3 of the post-alcohol bard loaded in the corresponding period (in the current day); x_2 is the weight in 1000 kg of sugar beet pulp loaded in the corresponding period (in the current day); x_3 is the volume in м^3 of the gravel loaded in the corresponding period (in the current day); x_4 is the volume in м^3 of molasses loaded in the corresponding period (in the current day); x_5 is the humidity in %; x_6 is the temperature in ℃ of the fermentation environment.

The results of pH measurements in the interval form, which is due to the device measurement error of 1%, and the values of the influencing factors $x_j, j = 1, \ldots, 6$, are given in Table 1.

Table 1. Results of experimental measurements

i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	$x_{i,4}$	$x_{i,5}$	$x_{i,6}$	y_i^-	y_i^+
1	130	129	0	0	96.2	44.6	8.062	8.258
2	70	159	0	0	96.3	44.3	8.062	8.258
3	80	147	0	14.4	96.1	43.8	8.1318	8.329
4	130	120	39.4	10	96.782	43.3	8.102	8.298
5	200	101	0	0	96.8	42.9	7.933	8.126
6	150	97.5	16.5	0	96.737	42.8	8.102	8.298
7	30	102	14.4	10	97	42.5	8.24	8.44
8	110	112.5	14.4	15	96.8	42.3	8.013	8.207
9	320	69	0	5	97	42.1	8.102	8.298
10	120	100	14.4	0	96.9	42.2	8.052	8.248
11	130	0	0	0	96.3	41.6	7.973	8.167
12	210	37.5	14.4	0	96.2	40.6	8.082	8.278
13	220	51	14.4	0	96.7	39.6	7.963	8.157
14	40	33	0	11	96.2	38.7	8.072	8.268
15	220	137.5	14.4	10	96.8	38.2	7.934	8.126

Accordingly, the dimension of the optimization problem is $m = 10, N = 15$. The MATLAB R2023b Update 6 environment was used for numerical experiments. The optimization was carried out based on the interior point algorithm (fmincon Global Optimization Toolbox function) [26]. The following settings of the criteria for stopping the optimization iterations were used for the experiments:

- $FunctionTolerance = 1 \cdot 10^{-6}$,
- $StepTolerance = 1 \cdot 10^{-6}$,
- $OptimalityTolerance = 1 \cdot 10^{-6}$,
- $MaxFunctionEvaluations = Inf$,
- $MaxIterations = Inf$.

For all models, parameters were identified without using the proposed stopping criterion and with the criterion in the form of (4). In both cases, the optimization was successful. Optimization results for both cases in the form of a vector of parameter estimates $\vec{\beta}$ of the nonlinear model (13), a vector of coefficient estimates $\vec{\alpha}$, and the value of the objective function $\delta(\beta)$:

- without using criterion (4):

$$\vec{\beta} = (8.1100, 3.6311, 36.8214, -11.4062, 1.7083, -4.5979, 9.9481, -3.1739, 9.8767, -5.2053),$$

$$\vec{\alpha} = (0.3427, 0.6412, 0.2031, 0.1449, 0.9460, 0.0922, 0.4590, 0.6130, 0.0499, 0.3719, 0.7066, 0.1488, 0.7669, 0.2686, 0.9616),$$

$$\delta(\beta) = 6.7824 \cdot 10^{-10},$$

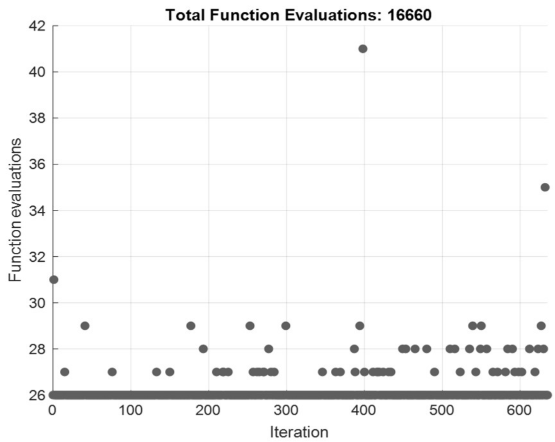
- using criterion (4):

$$\vec{\beta} = (8.0890, 1.4325, 16.8102, 10.3792, 1.1376, 10.3597, 9.9824, -1.8025, 9.9621, -2.4172),$$

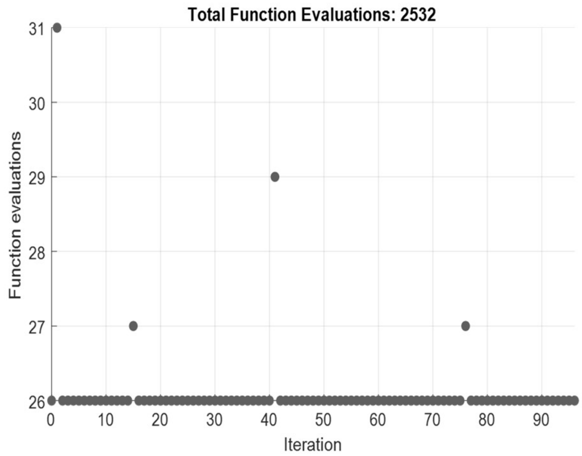
$$\vec{\alpha} = (0.4523, 0.9213, 0.2996, 0.1801, 0.9481, 0.1460, 0.0566, 0.6606, 0.0395, 0.4357, 0.6355, 0.1024, 0.7166, 0.3123, 0.9588),$$

$$\delta(\beta) = 2.0645 \cdot 10^{-4}.$$

Let's analyze the optimization results from the point of view of the number of iterations. Fig. 3 shows the number of calculations of the objective function value at each iteration during the optimization and their total number. As we can see from Figure 3, the number of iterations and calculations of the value of the objective function during the optimization based on the standard stop criteria was k=635 (Fig. 3(a)), and based on the additional proposed stop criterion – k=96 (Fig. 3(b)).



(a)

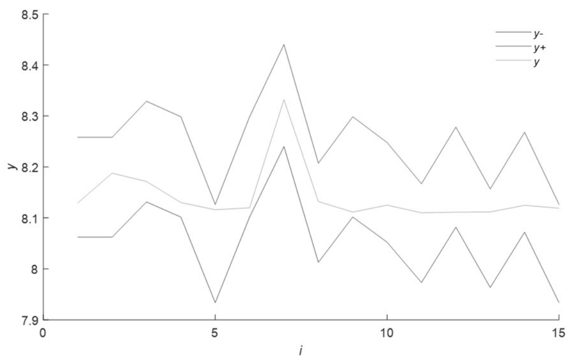


(b)

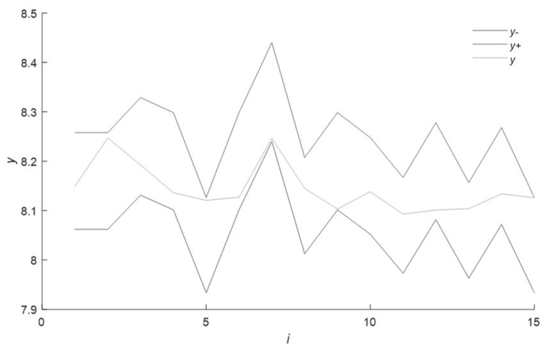
Figure 3. Number of iterations and calculations of the objective function value during optimization: (a) – based on standard stop criteria (k=635), (b) – using the proposed stop criterion (k=96).

Therefore, using the proposed stopping criterion made it possible to reduce the number of iterations during optimization by 6.6 times. From the point of view of evaluating the complexity in the form of the number of calculations of the objective function, the same ratio of 16660/2532 = 6.58 reduction of computational complexity is observed.

It is also worth noting that in both cases, the obtained interval models have "guaranteed" prognostic properties and reflect the properties of the static system with a specified accuracy. The models are adequate because the simulated values belong to the numerical intervals which obtained experimentally. It is illustrated in Fig. 4.



(a)



(b)

Figure 4. Graphs of model-based values and interval values of measurements: (a) – based on standard stop criteria, (b) – using the proposed stop criterion

In general, based on the conducted experiments, it was established that using the proposed criterion made it possible to reduce the number of iterations by 4.5 times, which is proportional to the same reduction in the number of calculations of the objective function (Table 2).

Table 2. Results of numerical experiments

Experiments number	The optimization problem dimension		Average iterations number		Efficiency, $\frac{k_{st\_cr}}{k_{new\_cr}}$
	m	N	Standard stop criteria, k_{st_cr}	Using the proposed stop criterion, k_{new_cr}	
50	10	15	16342	2302	7,1
50	10	50	25784	4159	6,2
50	10	100	37632	6969	5,4
50	20	50	28743	6387	4,5
50	20	100	41674	10967	3,8
50	30	50	31738	15113	2,1
50	30	100	48287	20120	2,4
Efficiency in general					4,5

Taking into account the experimentally obtained results, it can be asserted that using the proposed criterion based on expression (4) in the optimization problem (6)-(8) provides a reduction of the computational complexity of the parametric identification method. Asymptotic estimates of the complexity of the parametric identification method, taking into account the experimentally confirmed efficiency, are shown in Fig. 5.

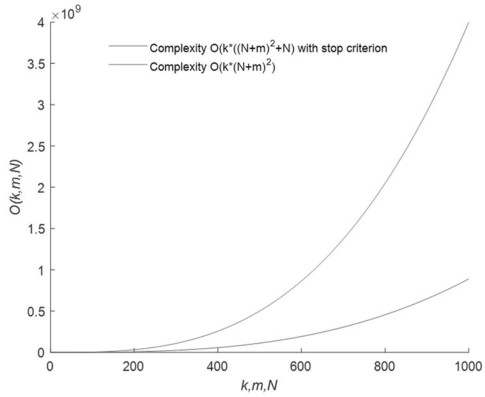


Figure 5. Asymptotic evaluations of the computational complexity of the parametric identification method based on the proposed stopping criterion

It should also be noted that the obtained results are important for the problems of structural identification of interval nonlinear models, where the selection of the model structure is based on the evaluation of the parameters of candidate models [11].

IV. CONCLUSIONS

The paper analyzed the computational complexity of the parametric identification method for interval nonlinear models of static systems based on the expansion of the parameter space. The method is based on the solution of an optimization problem with a smooth objective function in the form of a convolution of interval constraints by nonlinear optimization methods, in particular, the interior point method. In practice, it was found that the optimization has excessive complexity, which is caused by a large number of iterations when approaching the optimal value $\delta(\vec{\beta}) = 0$ of the objective function.

Taking into account the considerations of the interval approach, it is proposed to use the optimization stop criterion based on the determination of model adequacy at the current iteration of the parameter identification method. The use of the proposed criterion provided a reduction in the number of iterations by 4.5 times, which is proportional to the same reduction in the number of calculations of the objective function. At the same time, the obtained nonlinear models have guaranteed prognostic properties and reflect the properties of the static system with a specified accuracy. It is worth noting that the considered optimization method can be applied only in the case of a smooth objective function.

The obtained results are important for the problems of structural identification of interval nonlinear models of static systems, where the selection of the model structure is based on the evaluation of the parameters of candidate models.

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