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Modeling Risk Exposure: Fuzzy and Fuzzy Intuitionistic Approaches to Pedestrian and Vehicle Interaction

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ABSTRACT Road safety is a major concern that raises significant worries, especially regarding accidents involving pedestrians. Often, the study of interaction between pedestrians and vehicles focuses on various measurable factors such as vehicle speed and pedestrian crossing speed, often overlooking human behaviors that have a significant impact on this interaction. In this regard, studying road risks poses a challenge that requires a systematic approach to successful overcoming. In this article, we compare both fuzzy and intuitionistic approaches to assess pedestrians' exposure to accident risks. These two approaches take uncertainty into account in a more natural way than classical methods based on precise values. Being more adept at handling uncertainty than classical methods, these approaches provide a finer understanding of reality, thus enabling the development of more tailored safety measures to protect pedestrians. Comparative analysis of the results highlights a significant improvement in the accuracy of risk assessments, underscoring the effectiveness of these approaches in the context of road safety.

KEYWORDS intuitionistic fuzzy number; accident risk; risk exposure indicator; pedestrian-vehicle interaction; simulation

I. INTRODUCTION

WHETHER we are pedestrians or drivers, we can all fall victim to various inherent driving dangers. However, pedestrians consistently remain the primary victims of road accidents, with drivers often being the main culprits of committing offenses, given that pedestrians are more vulnerable to road accidents due to their lack of physical protection. In the event of a collision with a vehicle, the consequences can be severe for pedestrians. In this context, human behaviors play a crucial role in collision risks. Errors and reckless behaviors of both drivers and pedestrians can contribute to an increased risk of accidents.

The absence of adequate signage, particularly on unsignposted crosswalks, can create situations of conflict between pedestrians and drivers, with each hoping to go first, thus increasing the risk of collision. Some drivers may be attentive or may even yield the right of way to pedestrians, while others may be inattentive or fail to respect the rules of pedestrian priority. Using a cell phone, manipulating electronic devices or any other activity that diverts the driver's attention from the road can lead to accidents; in other words, distracted driving is one of the main causes of road accidents. What is more, disregarding speed limits considerably increases the risk of accidents. Excessive speed reduces drivers' reaction time and makes it more difficult to control the vehicle, which can lead to collisions.

Pedestrians also have a role to play in accident prevention. Crossing the road carelessly, without respecting crosswalks or checking for vehicles, can considerably increase the risk of collisions. Using cell phones or headphones while crossing the road can also distract pedestrians and make them less aware of their surroundings. It is essential that pedestrians adopt a responsible attitude and remain vigilant when on foot.

For this reason, it is crucial to analyze the interaction between pedestrians and vehicles to reduce the risk of accidents involving pedestrians. By understanding how pedestrians and drivers interact on the road, we can put in place appropriate safety measures to prevent accidents.

The plan of our paper revolves around several crucial steps for a thorough understanding of accident risk modeling. First, we begin our exploration by defining the fundamental concepts of fuzzy logic and intuitionistic fuzzy logic. This first section aims to establish a solid foundation for our methodological approach. Next, we will dive into the definition of pedestrian and vehicle dynamic's models, sketching the contours of these elements without going into specific details. This step is crucial to the implementation of our simulation prototype.

The third section of our paper will focus on modeling risk through fuzzy approach, exploiting the principles of fuzzy set theory to deal with the uncertainty associated with risk factors. Having examined this approach, we will go a step further by introducing intuitionistic fuzzy logic into our risk modeling. This section will highlight how the addition of intuitionism enriches our understanding by taking into account the subjective aspects of risk assessments.

Finally, the last section of our paper will focus on the results of risk modeling through simulations. This practical component will validate the effectiveness of our approach by providing concrete data on accident risk management. Through this structure, our paper aspires to make a significant contribution to accident risk modeling.

II. RELATED WORKS

Given the number of accidents occurring at unsignalized crosswalks, much research has been carried out on the interaction between pedestrians and vehicles. In this section, we will explore some of these studies, focusing on the approaches used to measure and assess risk.

In order to study the severity of pedestrian-vehicle interactions on a database of 2954 interactions, Govinda et al., adopted multilinear regression (MLR) which was developed using SPSS software and taking into account pedestrian age, gender and speed, vehicle type, P-V interaction position, interaction location and crossing type. On the other hand, machine learning via the support vector machine (SVM) method was approved to estimate threshold values of the risk indicator (RI) for various pedestrian and vehicle characteristics [1].

X. Shen and P. Raksincharoensak proposed a statistical framework for assessing the risk of vehicles crossing an unsignalized intersection. First, an intensity model of the near miss event is established by considering the near miss event as a non-homogeneous Poisson process. The non-homogeneous Poisson process is defined on the sigma-algebra of the two-dimensional plane of vehicle speed and distance to the intersection, rather than on the time axis. On the other hand, pedestrian intention is defined as a binary variable with 1 for crossing and 0 for stopping. The logistic function is applied to model the probability of pedestrian intention. Based on the residual analysis, the risk model is established to give a quantitative predictive risk measure when pedestrians appear [2].

Wu et al. proposed a Bayesian DBN (Dynamic Bayesian Network) that combines continuous observable variables collected by vehicle sensors such as: pedestrian speed orientation, the lateral distance between pedestrian and vehicle roadside, and longitudinal distance between vehicle and pedestrian and discrete variables hidden in pedestrians' minds such as: pedestrian's decision to cross, pedestrian's feeling of danger when crossing), then an estimation of pedestrian's trajectory by probabilistic reasoning. For a risk assessment, they developed the DSF (Driving Safety Field), a road safety field model based on the pedestrian's predicted trajectory [3].

Ezzati et al. used substitution safety measures (SSMs) to identify future accident outcomes and formulate a model capable of predicting the threats of pedestrian-vehicle conflict. Conflict thresholds are determined using three methods: intersection point, p-tile, maximum variability between classes and minimum entropy [4].

III. FUZZY LOGIC

Fuzzy logic, also known as fuzzy set theory, is a field of mathematics and computer science that was developed by the American-Iranian mathematician and electrical engineer Professor Lotfi Zadeh in 1965. In his seminal paper "Fuzzy Sets", Zadeh introduced fuzzy logic by proposing to replace the traditional binary logic of "true or false" with a logic that allows degrees of truth, rather than binary values, to be represented and manipulated [5].

Fuzzy logic, based on the theory of fuzzy sets, is proving to be a powerful approach for modeling situations characterized by gradual degrees of truth.

This conceptual framework is based on the representation of variables and relations as membership functions, offering valuable flexibility in the management of uncertainty. By using fuzzy sets, fuzzy logic makes it possible to describe concepts that are not clearly defined, which is particularly useful for decision-making in complex, dynamic environments.

This method has proved its effectiveness across a wide range of industrial fields, where it has been successfully adopted to provide accurate results adapted to constantly changing contexts. In the field of accident risk studies, fuzzy logic has seen widespread use due to its ability to deal with the uncertain and imprecise information that often characterizes these situations. By enabling the formal representation of variability and ambiguity, fuzzy logic is positioned as an essential tool for better understanding and managing risk factors, making a significant contribution to improving accident prevention and management strategies.

A. DEFINITION: FUZZY SET

A fuzzy set A in a universal set X is defined by a membership function $\mu_A(x)$ which assigns to each element x of X a value in the interval [0,1]. This value represents the degree to which x belongs to the fuzzy set A [6,7].

Formally, a fuzzy set A in X is defined as follows:

$$A = \{(x, \mu_A(x)) | x \in X, \mu(x) \in [0,1] \}, \tag{1}$$

where x is an element of the universal set X and $\mu_A(x)$ is the membership function of A.

B. DEFINITION: TRIANGULAR FUZZY NUMBER

A triangular fuzzy number is a fuzzy number $\tilde{A}$ defined by a triplet $tfn(a_1, a_2, a_3)$, having membership function as follows:

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \le x < a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 < x \le a_3 \\ 0 & a_3 < x \end{cases}, \tag{2}$$

where a_1, a_2 and a_3 are the lower bound, midpoint, and upper bound of the fuzzy number $\tilde{A}$ [8].

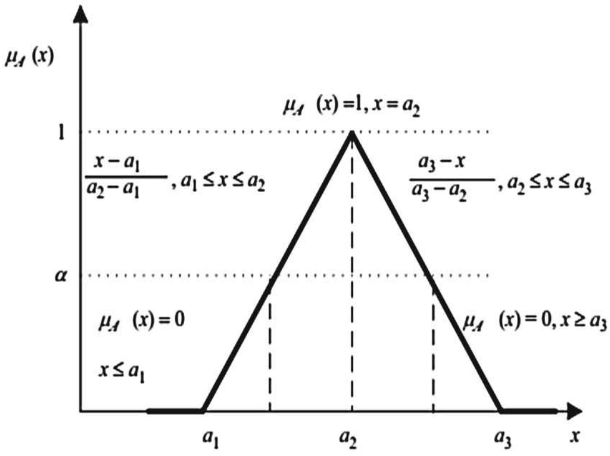

Figure 1. Triangular Fuzzy Number representation

C. ARITHMETIC OPERATIONS

Let $\tilde{A}=tfn(a_1,a_2,a_3)$ and $\tilde{B}=tfn(b_1,b_2,b_3)$ be triangular fuzzy numbers, the arithmetic of triangular fuzzy numbers boils down to direct application of arithmetic operations on bounds and median, the table below summarizes the operations of triangular fuzzy numbers:

Table 1. Operations of triangular fuzzy numbers

Operation	Results
Addition	$\tilde{A}+\tilde{B}=tfn(a_1+b_1,a_2+b_2,a_3+b_3)$
Subtraction	$\tilde{A}-\tilde{B}=tfn(a_1-b_3,a_2-b_2,a_3-b_3)$
Multiplication	$\tilde{A}\times\tilde{B}=tfn(a_1\times b_1,a_2\times b_2,a_3\times b_3)$
Division	$\tilde{A}/\tilde{B}=tfn(a_1/b_3,a_2/b_2,a_3/b_1)$

Multiplication and division of two fuzzy triangular numbers depend on their sign, especially when positive or negative numbers are involved. For positive triangular fuzzy numbers, the operation is generally performed by multiplying or dividing the components of the lower, middle and upper bounds. The table above summarizes the cases of multiplication and division of two positive fuzzy triangular numbers [9].

IV. INTUITIONISTIC FUZZY LOGIC

Intuitionistic logic, developed mainly by the Russian mathematician Andrei Kolmogorov and the Dutch philosopher Luitzen Brouwer in the early 20th century, focuses on the concept of constructive truth. It rejects the principle of the excluded third, asserting that a proposition can be either true or false, but not necessarily one or the other. It also emphasizes constructive proof, where a proposition is considered true only if an actual proof of its truth exists.

Intuitionistic fuzzy logic combines these two approaches by integrating the degrees of truth of fuzzy logic with intuitionistic logic. This means that, in an intuitionistic fuzzy context, a proposition can have a fuzzy degree of truth and the truth can be established constructively, i.e., it can be deduced from an actual proof. Intuitionistic fuzzy logic is an extension of traditional fuzzy logic, introducing an additional dimension to take account of the degree of non-membership [6, 7].

This approach is often used to model human reasoning in situations where knowledge is incomplete or ambiguous. It is also integrated into decision support systems to deal with uncertain information and to provide recommendations based on degrees of confidence. In addition, intuitionistic fuzzy logic can be effectively applied in the field of road safety, where it can be used to model traffic data.

A. DEFINITION: INTUITIONISTIC FUZZY SET

An intuitionistic fuzzy set A in a universal set X is defined by a membership function $\mu_A(x)$ and a non-membership function $\vartheta_A(x)$ which assign to each element x of X a value in the interval [0,1].

Formally, an intuitionistic fuzzy set A in X is defined as follows:

$$A=\{(x,\mu_A(x),\vartheta_A(x))|x\in X\}, \quad (3)$$

where:

$$\mu_A(x): X\to[0,1]$$

and

$$\vartheta_A(x): X\to[0,1]$$

are the degree of membership and the degree of non-membership of element x in X, such that:

$$0\le\mu_A(x)+\vartheta_A(x)\le 1 . \quad (4)$$

Furthermore, the value π=1-$\mu_A(x)$ - $\vartheta_A(x)$ is called the degree of uncertainty of x in X.

B. DEFINITION: INTUITIONISTIC TRIANGULAR FUZZY NUMBER

An intuitionistic triangular fuzzy number (ITFN) denoted by $\widetilde{A^\iota}=TIFN\left(\underline{a},a,\overline{a},\alpha_{\widetilde{A^\iota}},\beta_{\widetilde{A^\iota}}\right)$ is an intuitionistic fuzzy set in R whose membership function $\mu_{\widetilde{A^\iota}}(x)$ and non-membership function $\vartheta_{\widetilde{A^\iota}}(x)$ are defined as follows [6]:

$$\mu_{\widetilde{A^\iota}}(x)=\begin{cases}\frac{(x-\underline{a})\times\alpha_{\widetilde{A^\iota}}}{(a-\underline{a})} & \text{for } \underline{a}<x<a\\ \alpha_{\widetilde{A^\iota}} & \text{for } x=a\\ \frac{(\overline{a}-x)\times\alpha_{\widetilde{A^\iota}}}{(\overline{a}-a)} & \text{for } a<x<\overline{a}\\ 0, & \text{for } x<\underline{a} \text{ or } \overline{a}<x\end{cases}, \quad (5)$$

and

$$\vartheta_{\widetilde{A^\iota}}(x)=\begin{cases}\frac{(a-x)+(x-\underline{a})\times\beta_{\widetilde{A^\iota}}}{(a-\underline{a})} & \text{for } \underline{a}<x<a\\ \beta_{\widetilde{A^\iota}} & \text{for } x=a\\ \frac{(x-a)+(\overline{a}-x)\times\beta_{\widetilde{A^\iota}}}{(\overline{a}-a)} & \text{for } a<x<\overline{a}\\ 1, & \text{for } x<\underline{a} \text{ or } \overline{a}<x\end{cases}. \quad (6)$$

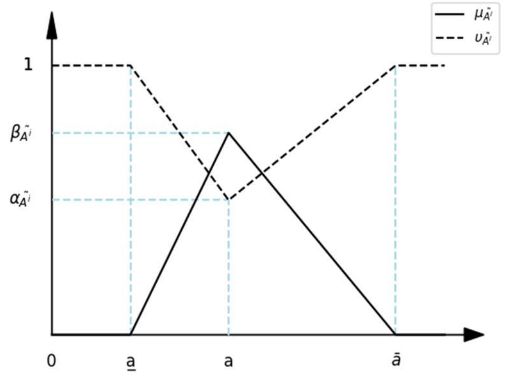

Figure 2. Triangular Intuitionistic Fuzzy Number representation

V. SIMULATING PEDESTRIAN AND VEHICLE DYNAMICS

Establishing a reliable simulation model to realistically represent the behavior of vehicles giving way and pedestrians crossing crosswalks is a crucial step in road safety and urban planning. This is because these delicate interactions between road users are critical points, where the risk of accidents is significant. Accurate simulation of these scenarios makes it possible to analyze and anticipate the behavior of drivers and pedestrians in a variety of contexts, thus contributing to the design of safer policies and infrastructures.

The simulation model needs to take into account a variety of factors, such as visibility, signage, priority rules, traffic density, and individual road user behavior. Integrating these elements helps to capture the complexity of interactions and decision-making that occur at crosswalks. Realistic modeling must also consider variations in road user characteristics, such as differences in pedestrian walking speeds, the types of vehicles involved, and environmental conditions.

A well-designed simulation offers the opportunity to evaluate the effectiveness of different crosswalk configurations, test signal strategies, and explore hypothetical scenarios to identify points of vulnerability. This information can be crucial for urban planners, traffic engineers and road safety managers to make informed decisions in the design of intersections, pedestrian zones and traffic policies.

A. FUZZY PEDESTRIAN MODEL

Having integrated the ant colony optimization metaheuristic (ACO) into the pedestrian simulation, we rely on a bio-inspired approach that takes advantage of the careful observation of ants' collective foraging behavior. ACO can be applied to realistically simulate pedestrian movements in a specific environment using algorithms inspired by ant biology [12, 13].

Just as ants communicate with each other through chemical signals left on their path called "pheromones", pedestrians in the simulation can be represented as entities interacting with each other and with their environment. ACO algorithm models pedestrians' individual choices according to factors such as proximity to other pedestrians, obstacles on the path, and the search for an optimal path to their destination. The advantage of this model lies in its ability to capture emergent behaviors at the collective level, while taking into account local interactions between pedestrians. This approach can be particularly valuable for understanding how pedestrians react to changes in their environment, such as the appearance of new obstacles, changes in pedestrian signage, or crowd density.

Moreover, having integrated fuzzy logic into the pedestrian model, we have enriched the simulation by introducing a further level of sophistication into pedestrian decision-making. Fuzzy logic makes it possible to model more realistically the complexity of factors influencing pedestrian choices, taking into account often imprecise and subjective nature of the information available. This could include the design of crossings to minimize potential conflicts between pedestrians and vehicles, the planning of more efficient pedestrian routes, or the design of public spaces to encourage smooth and safe pedestrian traffic. See Boulmakoul, A., Mandar, M. [12].

B. VEHICLE MODEL

We used the Nagel-Schreckenberg model, a theoretical mathematical model designed to simulate the behavior of road traffic on a single-lane road. This model, developed in the early 1990s by German physicists Kai Nagel and Michael Schreckenberg, is a simple cellular automaton dedicated to road traffic flows. Its ability to reproduce traffic jams, by slowing down the average speed of vehicles when the road is congested, makes it an essential tool in the study of traffic-related phenomena [14].

The parameters taken into account by this model include maximum speed, braking, acceleration and overtaking rules, enabling realistic simulation of vehicle movements on a road. The use of the Nagel-Schreckenberg model has proved invaluable in the analysis of road traffic properties such as traffic density, average vehicle speed, and the formation of traffic jams. The rules of the model are relatively simple, but they capture essential behaviors. Each vehicle has an initial speed, and at each stage, it can accelerate if it can do so within the speed limit. The vehicle can also brake to avoid a collision with the vehicle in front. A random deceleration rule is often incorporated to represent elements of realistic behavior. See Kai Nagel, Michael Schreckenberg [14].

VI. RISK MODELLING

Exposure to the risk of accidents between pedestrians and vehicles is intrinsically linked to factors such as pedestrian crossing times, vehicle braking times and road user behavior. Pedestrian and driver behaviors also play a crucial role. Factors such as distraction, disregard for traffic rules, and inappropriate speed can increase the risk of accidents. The time it takes for a pedestrian to cross an intersection is influenced by several parameters, including walking speed, traffic density, and roadway configuration.

Complex crosswalks or poorly synchronized traffic lights can lead to short crossing times, which can increase the risk of accidents, especially in high-traffic areas, where drivers have less time to react.

On the other hand, vehicle braking time is crucial to avoid collisions with pedestrians. It depends on vehicle speed, weather conditions, road conditions, and driver responsiveness. Longer braking times may be necessary in emergencies, such as the sudden presence of a pedestrian on the road. Advanced technologies such as emergency braking systems can help reduce these reaction times and minimize the risk of accidents.

Understanding and managing exposure to the risk of accidents between pedestrians and vehicles therefore requires precise assessment of pedestrian crossing times and vehicle braking times.

A. FUZZY RISK MODELING

A.1 OLD RISK EXPOSURE FORMULA

According to Cohen (1993), safety distance is defined as follows:

$$S = L + T_r v + \frac{v^2}{2\gamma}, \quad (7)$$

where L is the car's length, v its speed, γ its acceleration and T_r is the driver's reaction time.

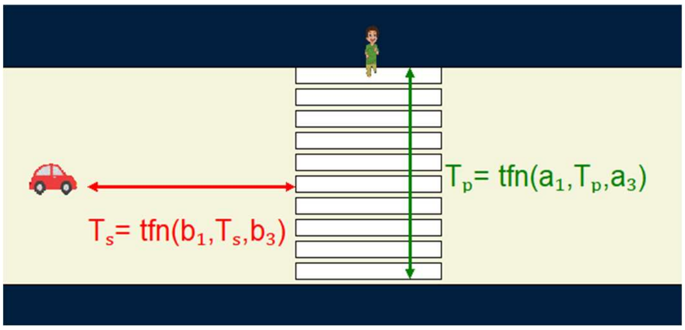

Figure 3. Pedestrian-Vehicle Interaction

By dividing the safety distance by the vehicle speed we obtain the safety stopping time T_s :

$$T_s = \frac{S}{v} = \frac{L}{v} + T_r + \frac{v}{2\gamma}. \quad (8)$$

Based on the transformation of the above formula by the statistical study in France, we obtain:

$$T_s = T_r + \frac{v}{2\gamma}. \quad (9)$$

Also, vehicle speed can be expressed as a function of vehicle density:

$$v = v_{max}(1 - \frac{\rho}{\rho_{max}}). \quad (10)$$

Finally, T_s becomes:

$$T_s = T_r + \frac{v_{max}}{2\gamma}\left(1 - \frac{\rho}{\rho_{max}}\right). \quad (11)$$

On the other hand, pedestrian exposure to accident risk is defined by Mandar and Boulmakoul (2014) [15]:

$$E(t) = \int_0^{T_p} q_v \, dt = q_v . \, T_p , \quad (12)$$

where T_p is the pedestrian crossing timeand q_v is the vehicle flow, which can be defined as a function of flow and vehicle speed: $q_v = v.\rho$.

Replacing the speed by (10) and representing T_p by a triangular fuzzy number to better take into account the uncertainty and variability associated with this measure $\widetilde{T_p} = tfn(T_p, \alpha, \alpha)$, we obtain as a first formula of exposure to accident risk:

$$\tilde{E}(t) = \widetilde{T_p}.\rho.v_{max}\left(1 - \frac{\rho}{\rho_{max}}\right). \quad (13)$$

A.2 NEW RISK EXPOSURE FORMULA

A second risk exposure formula takes into account the safety time T_s which, like T_p, is represented by a triangular fuzzy number $\widetilde{T_s} = tfn(T_s, \alpha, \alpha)$:

$$\widetilde{E'} = (\widetilde{T_s} - \widetilde{T_p}).v.\rho = \widetilde{T_s}.q - \widetilde{T_p}.q . \quad (14)$$

Replacing $\widetilde{T_s}$ by (5), we obtain:

$$\widetilde{E'} = \left(T_r + \frac{v_{max}}{2\gamma}\left(1 - \frac{\rho}{\rho_{max}}\right)\right).q - \widetilde{T_p}.q . \quad (15)$$

By factoring by q, and posing: $\alpha = T_r - \widetilde{T_p}$ and $\beta = \frac{1}{2\gamma}$, the second risk exposure formula becomes:

$$\widetilde{E'} = \left(\alpha + \frac{v_{max}}{2\gamma}\left(1 - \frac{\rho}{\rho_{max}}\right)\right).q . \quad (16)$$

B. INTUITIONISTIC FUZZY RISK MODELING

Having already explored fuzzy logic, we are now going one step further by turning to intuitionistic fuzzy modeling to understand accident risk. Accidents are not simply the result of a single cause, but rather the convergence of multiple factors, each carrying its degree of uncertainty and subjectivity. These complex factors make accident risk modeling a particularly delicate task.

The methodology of intuitionistic fuzzy modeling is essential here, as it offers a robust approach capable of handling the complexity inherent in such data. By implementing fuzzy set theory and incorporating intuitive aspects of human cognition, this approach enables a more faithful representation of the nuances and imprecisions associated with risk assessments.

Thus, by embracing these aspects, intuitionistic fuzzy modeling becomes a key to better understanding the variability and indeterminacy surrounding accidental conditions, paving the way for more comprehensive and adaptive risk management strategies.

Once again, we represent T_p and T_s by triangular intuitionistic fuzzy numbers [7]:

$$\widetilde{T_p}^{\iota} = (\mu^d(x), \vartheta^d(x))$$

$$= TIFN(T_p - \sigma_d, T_p, T_p, +\sigma_d, \alpha_d, \beta_d), \quad (17)$$

where $\mu^d(x)$ indicates the degree of indecision available to the vehicle driver to correctly assess the pedestrian crossing time;

$\vartheta^d(x)$ indicates the degree of uncertainty available to the driver to misjudge the pedestrian crossing time.

$$\widetilde{T_s}^{\iota} = (\mu^p(x), \vartheta^p(x))$$

$$= \mathrm{TIFN}(T_s - \sigma_p, T_s, T_s + \sigma_p, \alpha_p, \beta_p), \quad (18)$$

where $\mu^p(x)$ indicates the degree of indecision the pedestrian has in correctly estimating the vehicle's safety time;

$\vartheta^p(x)$ indicates the degree of uncertainty the pedestrian has in correctly assessing the vehicle's safety stopping time.

Table 2 shows a risk assessment matrix based on the accuracy of the driver and pedestrian assessments. Each cell in the table reflects the degree of risk associated with matching or mismatching assessments. This refers to the ability of the driver and pedestrian to correctly perceive and interpret the traffic dynamics and intentions of the other party in a pedestrian-vehicle interaction.

On the one hand, the driver's assessment concerns his or her ability to recognize the presence of a pedestrian, anticipate the latter's movements and make decisions accordingly. This can include recognizing pedestrian signals, such as the intention to cross, and adjusting vehicle speed or trajectory accordingly.

On the other hand, pedestrian assessment involves the ability to correctly perceive traffic, identify safe opportunities to cross the road, and effectively communicate their intentions to drivers, for example, by making a clear gesture to indicate that they wish to cross.

Table 2. Risk Assessment Matrix for Pedestrian-Vehicle Interactions.

Driver assessment	Pedestrian assessment	Risk
$\mu^d(x)$: correct	$\mu^p(x)$: correct	μ^+
$\mu^d(x)$: correct	$\vartheta^p(x)$: incorrect	μv^{++}
$\vartheta^d(x)$: incorrect	$\mu^p(x)$: correct	μv^{+++}
$\vartheta^d(x)$: incorrect	$\vartheta^p(x)$: incorrect	v^{++++}

This risk matrix provides a systematic representation of road safety scenarios, highlighting situations with varying levels of risk associated with each combination of assessments. If the driver correctly assesses the situation and the pedestrian does the same, this can be considered a mutually correct assessment, leading to a low level of risk.

On the other hand, if either fails to assess the situation correctly, this leads to a mutual incorrect assessment, increasing the level of risk.

The four levels of risk represent a graduated scale for classifying the four situations according to their degree of danger, as follows:

i. μ^+ : Low Risk
ii. μv^{++} : Moderate Risk
iii. μv^{+++} : High Risk
iv. v^{++++} : Very High Risk

VII. RESULTS AND DISCUSSION

To model pedestrian behavior, we used the Ant Colony Optimization (ACO) model, described in Section V.A. Each pedestrian is represented in a cell surrounded by 8 neighboring cells, from which a destination cell is chosen after the ACO model has been run. On the other hand, in our simulation, vehicles are modeled according to the Nagel-Schreckenberg model, detailed in section V.B. Vehicles travel at speeds between 40 and 130 km/h, in accordance with the speed limits imposed on the roads. The simulation was carried out over a period of 120 seconds, on a road 50 cells long. These models were implemented in the Python programming environment, in particular using the Tkinter library. Tkinter facilitated the creation of an interactive graphical interface for the simulation, enabling visual representation of pedestrian-vehicle interaction scenarios. In fact, the parameters included in the risk exposure formulas were extracted from the simulation in order to calculate these formulas. The results of the simulation were captured through four evocative graphs. Each of these graphs represents a distinct facet of the comparative analysis between the old and new formulations of fuzzy logic and intuitionistic fuzzy logic. These graphs aim to reveal the subtleties of risk exposure in realistic traffic situations, where the degrees of indecision and uncertainty of drivers and pedestrians play a crucial role.

A. FUZZY LOGIC GRAPHS

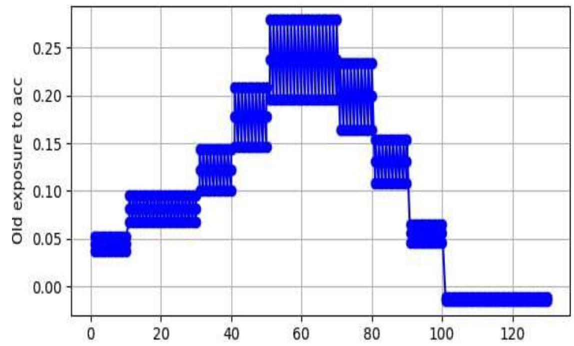

Figure 4. Old formulation of pedestrian accident risk indicator using Fuzzy Logic

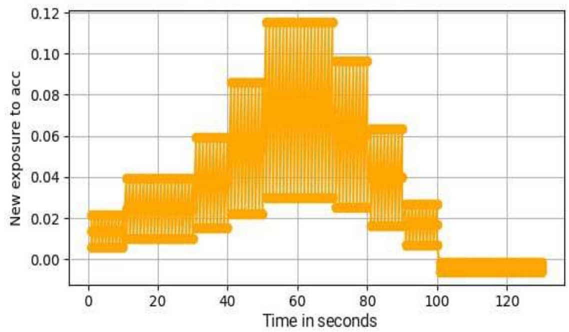

Figure 5. New formulation of pedestrian accident risk indicator using Fuzzy Logic

The first three graphs highlight the variations in the assessment of risk exposure according to fuzzy logic. The first chart shows the old formulation, highlighting areas of vagueness and uncertainty. The second chart shows the updated version, incorporating more precise and detailed concepts.

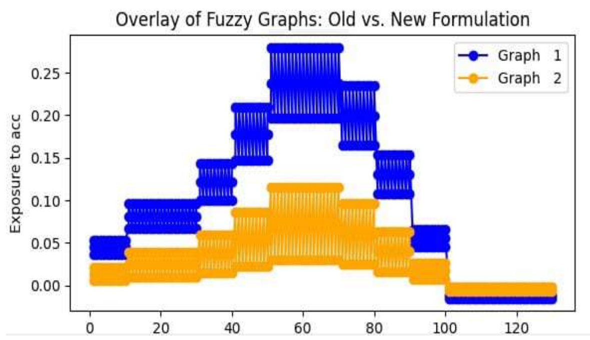

Figure 6. Overlay of the two Fuzzy graphs

This visual comparison enables observers to quickly grasp the improvements and refinements made to our modeling approach, offering a more nuanced and accurate representation of risk assessment. These visual adjustments provide important insights into the progress of our model, reinforcing its validity and ability to more accurately represent the complex nuances associated with risk exposure.

B. INTUITIONIST FUZZY LOGIC GRAPHS

The next three graphs represent an in-depth exploration of intuitionistic logic, bringing a different dimension to our risk analysis. The third graph illustrates the initial formulation, highlighting the distinctive features of intuitionistic logic. In this representation, the emphasis is on the gradation of risk understanding and acceptance, suggesting a more nuanced and progressive approach than that of fuzzy logic.

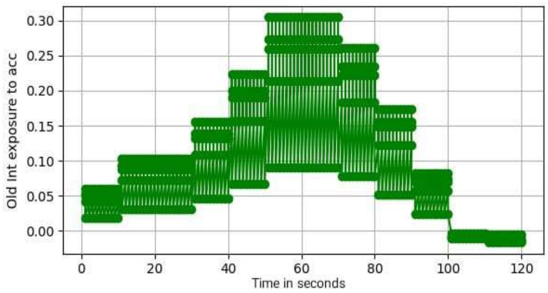

Figure 7. Old formulation of pedestrian accident risk indicator using Intuitionistic Fuzzy Logic

In comparison, the fourth graphic (Figure 7) shows the modifications introduced in the new approach based on intuitionistic logic. These visual adjustments reflect the nuances specific to this logic, highlighting the areas where gradual understanding has influenced our assessment of risk. By focusing on how risks are gradually understood and accepted, intuitionistic logic makes important distinctions from fuzzy logic, highlighting aspects often overlooked in more traditional models.

This visual exploration offers an instructive comparison between the two approaches, highlighting the distinctive features of intuitionistic logic in the context of our risk modeling.

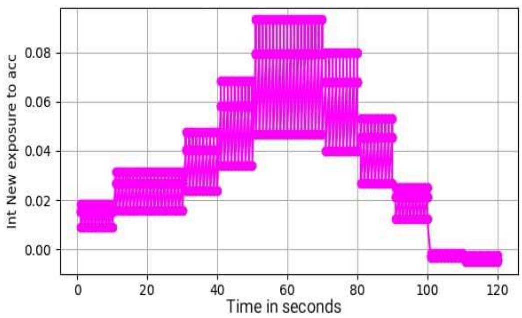

Figure 8. New formulation of pedestrian accident risk indicator using Intuitionistic Fuzzy Logic

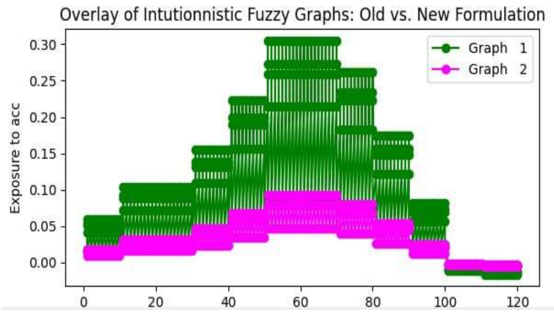

Figure 9. Overlay of the two Intuitionistic Fuzzy graphs

C. COMPARATIVE ANALYSIS

By examining these graphs comparatively, we can deduce the implications of the adjustments made to our models. Changes in the assessment of risk exposure, whether through fuzzy or intuitionistic logic, underline the importance of taking into account the complexity inherent in this field. This comparative approach provides a sound basis for discussing the advantages and disadvantages of both logics in the specific context of our study.

By carefully comparing the results generated by the two formulations, it becomes clear that the new formula, whether based on fuzzy or intuitionistic logic, offers a finer granularity in risk assessment than the old one. This finesse manifests itself in the increased ability to discern subtle variations in risk exposure levels. With fuzzy logic, the boundaries between risk categories are less abrupt, enabling a more detailed representation of ambiguous situations. Similarly, intuitionistic logic adds an extra dimension by capturing gradual evolutions in risk perception. Thus, the new formula, whether based on fuzzy or intuitionistic logic, proves to be a more refined tool for modeling the complexity inherent in risk exposure, offering a more nuanced and accurate view of this complex dynamic.

In conclusion, the combined use of fuzzy logic and intuitionistic logic offers a comprehensive and nuanced view of risk exposure, highlighting the dynamic and evolving aspects of this complex phenomenon.

VIII. CONCLUSION

In summary, our in-depth exploration of risk exposure assessment, exploiting both fuzzy logic and intuitionistic logic, highlights notable differences between the old and new formulations. Visual analysis through graphic overlay offers an instructive comparative perspective, highlighting the adjustments made by the new approach. Particularly noteworthy is the fact that the updated formula, whether based on fuzzy or intuitionistic logic, generates more precise risk values, offering a more detailed representation of the subtleties inherent in risk exposure.

References

[1] L. Govinda, M. R. SaiKiranRaju, K. V. R. Ravi Shankar, "Pedestrian-vehicle interaction severity level assessment at uncontrolled intersections using machine learning algorithms," *Safety Science*, vol. 153, pp. 1-9, 2022. https://doi.org/10.1016/j.ssci.2022.105806.

[2] X. Shen and P. Raksincharoensak, "Pedestrian-aware statistical risk assessment," *IEEE Transactions on Intelligent Transportation Systems,* vol. 23, no. 7, pp. 7910-7918, 2022. https://doi.org/10.1109/TITS.2021.3074522.

[3] R. Wu, X. Zheng, Y. Xu, W. Wu, G. Li, Q. Xu, Z. Nie, "Modified driving safety field based on trajectory prediction model for pedestrian–vehicle collision," *Sustainability*, vol. 11, 6254, 2019. https://doi.org/10.3390/su11226254.

[4] R. E. Amini, K. Yang, C. Antoniou, "Development of a conflict risk evaluation model to assess pedestrian safety in interaction with vehicles," *Accident Analysis & Prevention*, vol 175, pp 1-16, 2022. https://doi.org/10.1016/j.aap.2022.106773.

[5] L A. Zadeh, "Fuzzy sets," Inform Control, vol. 8, issue 3, pp. 338-353, 2014. https://doi.org/10.1016/S0019-9958(65)90241-X.

[6] K. T. Atanassov, "Intuitionistic fuzzy sets," in Intuitionistic Fuzzy Sets. *Studies in Fuzziness and Soft Computing*, vol. 35, pp. 1-137, 1999. https://doi.org/10.1007/978-3-7908-1870-3_1.

[7] M. Mandar, L. Karim, A. Boulmakoul, & A. Lbath, "Triangular intuitionistic fuzzy number theory for driver-pedestrians interactions and risk exposure modeling," *Procedia Computer Science*, vol. 109, pp. 148-155, 2017. https://doi.org/10.1016/j.procs.2017.05.309.

[8] M. A. Shafia and S. Abdollahzadeh, "Integrating fuzzy kano and fuzzy TOPSIS for classification of functional requirements in national standardization system," *Arabian Journal for Science and Engineering*, vol. 39, no. 8, pp. 6555–6565, 2014. https://doi.org/10.1007/s13369-014-1251-z.

[9] A. K. Mukherjee, K. H. Gazi, S. Salahshour, A. Ghosh, S. P. Mondal, "A brief analysis and interpretation on arithmetic operations of fuzzy numbers," *Results in Control and Optimization,* vol. 13, 100312, 2023. https://doi.org/10.1016/j.rico.2023.100312.

[10] P. G. Gipps, B. Marksjo, "A micro-simulation model for pedestrian flows," *Math Comput Simul*, vol. 27, pp. 95-105, 1985. https://doi.org/10.1016/0378-4754(85)90027-8.

[11] M. Pachter, P. R. Chandler, "Challenges of autonomous control," *IEEE Cont Syst Mag*, vol. 18, pp. 92-7, 1998. https://doi.org/10.1109/37.710883.

[12] A. Boulmakoul, M. Mandar, "Fuzzy ant colony paradigm for virtual pedestrian simulation," *The Open Operational Research Journal*, vol. 7, pp. 19-29, 2011. https://doi.org/10.2174/1874243201105010019.

[13] M. Dorigo, M. Birattari and T. Stutzle, "Ant colony optimization," *IEEE Computational Intelligence Magazine*, vol. 1, no. 4, pp. 28-39, 2006. https://doi.org/10.1109/MCI.2006.329691.

[14] K. Nagel, M. Schreckenberg, "A cellular automaton model for freeway traffic," *Journal de Physique I*, vol. 2, issue 12, pp. 2221-2229, 1992. https://doi.org/10.1051/jp1:1992277.

[15] M. Mandar, A. Boulmakoul, "Virtual pedestrians' risk modeling," *International Journal of Civil Engineering and Technology*, vol. 5, issue 10, pp. 32-42, 2014.

[16] W. H. K. Lam, J. Y. S. Lee, K. S. Chan, P. K. Goh, "A generalized function for modeling bi-directional flow effects on indoor walkways in Hong Kong," *Transp Res*, vol. 37, pp. 789-810, 2003. https://doi.org/10.1016/S0965-8564(03)00058-2.

[17] G. Ren, C. Gu, L. Lu, Z. Zhou, C. Ding, "Modeling risk degree of conflicts between crossing pedestrians and vehicles at signalized intersections," *J. Transp. Syst. Eng. Info. Technol*, vol. 12, issue 5, pp. 76–81, 2012. https://doi.org/10.1016/S1570-6672(11)60224-9.

[18] X. Sun, K. Lin, Y. Wang, S. Ma, H. Lu, "A study on pedestrian – Vehicle conflict at unsignalized crosswalks based on game theory," *Sustainability*, vol. 14, 7652, 2022. https://doi.org/10.3390/su14137652.

[19] L. Lu, et al., "A cellular automaton simulation model for pedestrian and vehicle interaction behaviors at unsignalized mid-block crosswalks," *Accid. Anal. Prev*, vol. 95, part B, pp. 425-437, 2016. https://doi.org/10.1016/j.aap.2016.04.014.

[20] P. Chen, C. Wu, S. Zhu, "Interaction between vehicles and pedestrians at uncontrolled mid-block crosswalks," *Safety Science*, vol. 82, pp. 68-76, 2016. https://doi.org/10.1016/j.ssci.2015.09.016.

[21] J. D. Fricker, & Y. Zhang, "Modeling pedestrian and motorist interaction at semi-controlled crosswalks: The effects of a change from one-way to two-way street operation," *Transportation Research Record*, vol. 2673, pp. 2-10, 2019. http.s://doi.org/10.1177/0361198119850142

[22] L. Yue, M. A. Abdel-Aty, Y. Wu and J. Yuan, "An augmentation function for active pedestrian safety system based on crash risk evaluation," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 11, pp. 12459-12469, 2020. https://doi.org/10.1109/TVT.2020.3017131.

[23] W. Zeng, H. Li, "Note on some operations on intuitionistic fuzzy sets," *Fuzzy Set, and Systems*, vol. 157, pp. 990- 991, 2016. https://doi.org/10.1016/j.fss.2005.08.007.

[24] J. Zhang, H. Wang, P. Li, "Cellular automata modeling of pedestrians crossing dynamics," *Journal of Zhejiang University - SCIENCE A*, vol. 7, pp. 835-40, 2004. https://doi.org/10.1631/jzus.2004.0835.

[25] B. J. Crowley-Koch, R. Van Houten, and E. Lim, "Effects of pedestrian prompts on motorist yielding at crosswalks," *Journal of Applied Behavior Analysis*, vol. 44, no. 1, pp. 121–126, 2011. https://doi.org/10.1901/jaba.2011.44-121.

[26] X. Liang, C. Wei, "An Atanassov's intuitionistic fuzzy multi-attribute group decision making method based on entropy and similarity measure," *International Journal of Machine Learning and Cybernetics*, vol. 5, issue 3, pp. 435-444, 2014. https://doi.org/10.1007/s13042-013-0178-0.

[27] B. R. Kadali, N. Rathi, and V. Perumal, "Evaluation of pedestrian mid-block road crossing behaviour using artificial neural network," *Journal of Traffic and Transportation Engineering (English Edition)*, vol. 1, pp. 111–119, 2014. https://doi.org/10.1016/S2095-7564(15)30095-7.

[28] U. Brude, and J. Larsson, "Models for predicting accidents at junctions where pedestrians and cyclists are involved. How well do they fit?" *Accident Analysis & Prevention*, vol. 25, pp. 499–509, 1993. https://doi.org/10.1016/0001-4575(93)90001-D.

[29] S. Okazaki, "A study of pedestrian movement in architectural space. Part 1: pedestrian movement by the application of magnetic models, "*Trans AIJ*, vol. 283, pp. 9-111, 1979. https://doi.org/10.3130/aijsaxx.283.0_111.

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