Date of publication JUN-30, 2023, date of current version JUN-19, 2023.
www.computingonline.net / computing@computingonline.net

Print ISSN 1727-6209
Online ISSN 2312-5381
DOI 10.47839/ijc.22.2.3091

Transformation of Mathematical Model for Complex Object in Form of Interval Difference Equations to a Differential Equation

MYKOLA DYVAK[1], ROMAN PASICHNYK[1], ANDRIY MELNYK[1], ANDRIY DYVAK[2], FRANK OTOO[1]
[1]West Ukrainian National University, 11, Lvivska street, Ternopil, 46009, Ukraine
(e-mail: mdy@wunu.edu.ua, roman.pasichnyk@gmail.com, melnyk.andriy@gmail.com, f.otoo@wunu.edu.ua)
[2]Ternopil Municipal Emergency Hospital,
2 Shpytalna Str, Ternopil, 46002, Ukraine, doriandre.ad96@gmail.com

Corresponding author: Mykola Dyvak (e-mail: mdy@wunu.edu.ua).

ABSTRACT Mathematical models of complex objects in the form of interval difference equations are built on the basis of the obtained experimental interval data within the limits of the inductive approach. At the same time, interpretation of physical properties of the object on the base of such model is complex enough. A method of transformation of a mathematical model in the form of interval differential equations was proposed in the article. The proposed method is based on the formulas for representing the values of the function at the node of the difference grid in the Taylor series in the neighborhood of the base node, as well as the differential representation of the derivatives in the same neighborhood. The developed approach creates opportunities for the identification of interval models of complex objects based on the analysis of interval data with further interpretation of the physical properties of the modeled object according to the classical scheme.

KEYWORDS interval differential equations; structural identification; interval model; Taylor series.

I. INTRODUCTION

RECENTLY, mathematical models of complex objects in the form of differential equations have become widely used for decision-making. Such equations are built on the basis of the obtained experimental data within the limits of the inductive approach [1-3]. At the same time, such models are often problematic for application because they should reflect important physical properties of the object based on the main modeling tasks [4-8]. When a deductive approach is used, the properties of the object are analyzed and then the corresponding mathematical equation is selected, which is solved and the solutions are compared with the results of the experiment [1-3]. Differential equations are the result of applying deductive approach for building mathematical models of the object for example for the purpose of displaying its dynamics. Simultaneously in this case, the coefficients of these equations reflect various properties of the object or phenomenon. Differential equations in partial derivatives in which coefficients also have physical content are also often used for modeling complex objects. For example, in the equations of mass transfer of particles of matter in air, these coefficients are diffusion coefficients [9-11]. Altogether, if mathematical models are built in the form of differential equations on the basis of experimental data using of the inductive approach, these coefficients do not have physical interpretation [12-16]. Therefore, the application of such type mathematical models for an investigation of object's properties particularly for other conditions, which coincide with conditions of getting experimental data is rather complex. It is also difficult to establish the adequacy of the model to the object under study [17-20]. Thus, such a contradiction arises that, on the one hand, the mathematical model of the object in the form of a difference equation, which is built on the basis of interval data, is consistent with experimental data, and on the other hand, it is difficult to assess the completeness of such model [21-25]. At the same time, advantages of obtaining mathematical model based on its identification using interval data are obvious, since the very process of model identification is solving an optimization problem using universal and well-developed methods [26-30]. One more advantage of such approach is that built mathematical models take into account errors in data on the base of application of interval data analysis [31-34]. In this case, mathematical model is built in the form of a difference equation, which ensures the calculation of object characteristic estimates in the form of numerical intervals [21, 34, 35]. It should be noted that optimization task of identification of such model is quite complex from mathematical point of view. However, for their solution enough methods have been developed based on swarm intelligence of a bee colony [21, 26-28]. In particular, the solution of optimization tasks based on behavioral model of a bee colony made it possible to solve a number of complex problems of mathematical modeling in ecology [15, 16,21]. Therefore, taking into consideration the known computing schemes of identification of mathematical models in the form of interval differential equations on the basis of experimental data, the development of a method of transformation of a mathematical model in the form of interval differential equation to mathematical model is appropriate. Such approach allows combining of positive sides of deductive and inductive approaches to building mathematical models and to their interpretation.

II. MATERIAL AND METHODS

The article is based on the materials of doctoral thesis. The authors took part in a number of research and development work.

Interval differential equations, structural identification, interval modeling is used for the experimental data processing and interpretation.

III. STATEMENT OF THE TASK

The task of identification of interval discrete model of complex object in the form of interval difference equation

$$v_k(\vec{V})=f_1(\vec{V})\cdot g_1+f_2(\vec{V})\cdot g_2+...+f_m(\vec{V})\cdot g_m, \quad k=d,...,K \tag{1}$$

based on interval data

$$[z_k^-;z_k^+], k=0,...,K, \tag{2}$$

is optimization task in the form of

$$\delta\left(\hat{\vec{g}}_l\right)\xrightarrow{\hat{\vec{g}}_l} min, \hat{\vec{g}}_l\subset\left[g_{jl}^{low};g_{jl}^{up}\right], \quad j=1,\ldots,m, l=1,\ldots,S, \tag{3}$$

where $[z_k^-;z_k^+]$ – numeric interval of obtained characteristic of an object experimentally with time or space discrete $k=0,...,K$; where v_k - means simulated value of a characteristic of an object in discrete moments of time (space) $k=d,...,K$; d – order of difference scheme (1); $\vec{g}$ – vector of parameters of a model, meanings of which should be assessed on the base of interval data; $f_1(\vec{V}), f_2(\vec{V}),..., f_m(\vec{V})$ - a set of basic functions, that is for each basic function, there will be in expression (1), the next vector:

$$\vec{V}=(v_{k-d},\ldots,v_{k-1},v_k)^T \tag{4}$$

An expression for function $\delta\left(\hat{\vec{g}}_l\right)$ is the following:

$$\delta\left(\hat{\vec{g}}_l\right)=\max_{i=1,\ldots,N}\left\{\left|mid\left(\left[\hat{v}_k\right]\right)-mid\left(\left[z_k^-;z_k^+\right]\right)\right|\right\}, \quad if\ [\hat{v}_k]\cap\left[z_k^-;z_k^+\right]=\varnothing, \exists k=0,\ldots,K \tag{5}$$

$$\delta\left(\hat{\vec{g}}_l\right)=\max_{i=1,\ldots,N}\left\{wid\left(\left[\hat{v}_k\right]\right)-wid\left(\left[\hat{v}_k\right]\cap\left[z_k^-;z_k^+\right]\right)\right\}, \quad if\ [\hat{v}_k]\cap\left[z_k^-;z_k^+\right]=\varnothing, \forall k=0,\ldots,K \tag{6}$$

Obtained difference equation is the following

$$[\hat{v}_k([\vec{\hat{V}}])]=[f_1([\vec{\hat{V}}])]\cdot\hat{g}_1+...+[f_m([\vec{\hat{V}}])]\cdot\hat{g}_m, \quad k=d,...,K, \tag{7}$$

where $[\vec{\hat{V}}]=[\hat{v}_{k-d}],...,[\hat{v}_{k-1}],[\hat{v}_k]$ - interval vector with components which mean computed interval estimates $[\hat{v}_k^-;\hat{v}_k^+]$ of characteristics.

It should be noted that stated above optimization task (3) is NP complex and algorithms of a bee colony (ABC) are used for its solution [16]. The main idea of ABC is to model behavior of a honey bee colony in the search of food (nectar).

Considering the context of a bee colony activity in nature, first, scout bees fly out of the hive looking for the nectar in a random direction. The quality of the nectar source is determined by the quantity of the nectar and also by a distance from the hive to the source of food. That is in a bee colony, there are some types of bees one group of which are scout bees. When the scout bees return to the hive, they inform other individuals of the colony about the found food sources. Worker bees choose the source of the nectar to which they will fly. This source is based on the received information about the quality of the found nectar. There is one more type of bees which are called explorer bees, the task of which is the search of the neighborhood of the nectar source. The main principle is the next: the better the food source is the more bees will fly to it. Then the process is repeated [16, 21].

It should be emphasized that in the context of computing of optimization task (3), the described principles of swarm intelligence can be formulated as follows [30]:

1. Initialization of the population of algorithm agents (in the space of search for solutions of the problem randomly) form a certain number of starting points (potential solutions of the optimization task (3)).
2. The movement of algorithm agents (based on a set of rules of movement specific to each swarm algorithm) agents move in the space of solutions of optimization task (3) in such a way as to approach the extremum of the objective function.
3. Completion of the procedure (when it is stopped, otherwise, it is transition to the second step).

It should be noted that nowadays, there are a lot of modifications of this algorithm particularly realized on parallel computations making this method the most effective for computing the stated above task among all swarm algorithms. That is we can state that inductive method for building mathematical models of dynamics of objects in the form of interval differential equations is enough effective and well developed. However, as it was stated above, built in the form of interval differential equations on the base of experimental data, mathematical models are complex for interpretation of physical properties of an object because these coefficients do not have physical interpretation.

Therefore, the research task is to develop a method, which gives an opportunity to transform this interval differential equation to interpret the properties of an object on the base of the built model and also to widen conditions of application of a model.

IV. METHOD OF TRANSFORMATION

As it is known, any differential operator can be represented by a difference equation with satisfactory accuracy. To build a transparent interpretation of an identified difference scheme based on experimental data, there is an opposite task that is to approach difference scheme with satisfactory accuracy using some differential equation. To solve it, the formulas for representing the values of the function at the node of the difference grid in the Taylor series in the vicinity of base node as well as the difference presentation of the derivatives in the same vicinity will be used. That is the same apparatus which is used for building difference schemes for a differential equation will be used for forming a differential equation which corresponds with a curtain accuracy to the given difference scheme. Let us formulate an algorithm for reducing a difference scheme to a differential equation followed by its investigation as follows:

1. The value of an unknown function at the node furthest from the base node is approximated. The value of the function is given at such a node using derivatives, the maximum order of which is determined by the distance of the analyzed node from the base one. The approximation process is continued for all nodes that do not coincide with the base node in the order of their distance from the latter one.
2. In one-dimensional case, the approximation process is continued for nodes that are more than two steps away from the base one. In order to save time of calculations, to approximate the value of the unknown function at nearby nodes, we use the widely used difference approximation equation of the first and second order derivatives.
3. By substituting the obtained equations into the difference scheme, we will obtain its approximation by a differential equation of the appropriate order.
4. A characteristic equation for the obtained differential equation of one variable or a sequence of characteristic equations with fixed values of all variables except one according to the method of straight lines are built.
5. Analytical presentations of solutions for characteristic equations are built. Those of them are selected which correspond to the nature of the task.
6. A general presentation of the solution of one-dimensional problem or a sequence of profiles of the solution of the problem in the case of many variables based on the boundary conditions of the problem is built.

Parameters of such analytical solutions, as a rule have transparent interpretation in the conditions of the given task. In addition, the solution to a differential equation connects to the task a number of similar processes of different nature, the dynamics of which are described by the same differential equation.

The peculiarities of the described approach will be explained on a separate example. Let us consider difference scheme for function of two variables of the next type:

$$v_{i,j} = g_1 + g_2 v_{i,j-1} + g_3 v_{i-1,j} + g_4 v_{i-1,j-1} + g_5 v_{i,j-2} + g_6 v_{i,j-3}. \quad (8)$$

The furthest from the base node *(i, j)* in the given scheme is a node *(i, j-3)*. That is why we begin from approximation of values of unknown function, it is in this node:

$$v_{i,j-3} = v(x_i, y_j - 3\Delta y) = v_{i,j} - \frac{\partial v}{\partial y}(i,j)\cdot(3\Delta y) + \frac{\partial^2 v}{\partial y^2}(i,j)\cdot\left(\frac{9}{2}\Delta y^2\right) - \frac{\partial^3 v}{\partial y^3}(i,j)\cdot\left(\frac{9}{2}\Delta y^3\right) + O(\varepsilon^4), \quad (9)$$

where $\varepsilon = \sqrt{\Delta x^2 + \Delta y^2}$.

Since it is planned to use the cubic derivative from this equation, we extend the expansion to a component per unit of higher order to ensure approximation properties. Gradually approximating, the following is obtained:

$$v_{i,j-2} = v(x_i, y_j - 2\Delta y) = v_{i,j} - \frac{\partial v}{\partial y}(i,j)\cdot(2\Delta y) + \frac{\partial^2 v}{\partial y^2}(i,j)\cdot(2\Delta y^2) - \frac{\partial^3 v}{\partial y^3}(i,j)\cdot\left(\frac{4}{3}\Delta y^3\right) + O(\varepsilon^4) \quad (10)$$

$$v_{i-1,j-1} = v(x_i - \Delta x, y_j - \Delta y)\ v_{i,j} - \frac{\partial v}{\partial x}(i,j)\cdot\Delta x - \frac{\partial v}{\partial y}(i,j)\cdot\Delta y + \frac{\partial^2 v}{\partial x^2}(i,j)\cdot\left(\frac{\Delta x^2}{2}\right) + \frac{\partial^2 v}{\partial x\partial y}(i,j)\cdot(\Delta x\Delta y) + \frac{\partial^2 v}{\partial y^2}(i,j)\cdot\left(\frac{\Delta y^2}{2}\right) - \frac{\partial^3 v}{\partial x^3}(i,j)\cdot\left(\frac{\Delta x^3}{6}\right) - \frac{\partial^3 v}{\partial x^2\partial y}(i,j)\cdot\left(\frac{1}{2}\Delta x^2\Delta y\right) - \frac{\partial^3 v}{\partial x\partial y^2}(i,j)\cdot\left(\frac{1}{2}\Delta x\Delta y^2\right) - - \frac{\partial^3 v}{\partial y^3}(i,j)\cdot\left(\frac{\Delta y^3}{6}\right) + O(\varepsilon^4), \quad (11)$$

$$v_{i-1,j} = v(x_i - \Delta x, y_j) = v_{i,j} - \frac{\partial v}{\partial x}(i,j)\cdot\Delta x + \frac{\partial^2 v}{\partial x^2}(i,j)\cdot\left(\frac{\Delta x^2}{2}\right) - \frac{\partial^3 v}{\partial x^3}(i,j)\cdot\left(\frac{\Delta x^3}{6}\right) + O(\varepsilon^4), \quad (12)$$

$$v_{i,j-1} = v(x_i, y_j - \Delta y) = v_{i,j} - \frac{\partial v}{\partial y}(i,j)\cdot\Delta y + \frac{\partial^2 v}{\partial y^2}(i,j)\cdot\left(\frac{\Delta y^2}{2}\right) - \frac{\partial^3 v}{\partial y^3}(i,j)\cdot\left(\frac{\Delta y^3}{6}\right) + O(\varepsilon^4) \quad (13)$$

Substituting equations (9) – (13) in presentation of difference scheme (8) the following is obtained:

$$v_{i,j} = g_1 + \propto_2 v_{i,j} + \propto_3 \frac{\partial v}{\partial x}(i,j)\cdot\Delta x + \propto_4 \frac{\partial v}{\partial y}(i,j)\cdot\Delta y + \propto_5 \frac{\partial^2 v}{\partial x^2}(i,j)\Delta x^2 + \propto_6 \frac{\partial^2 v}{\partial x\partial y}(i,j)\cdot\Delta x\Delta y + \propto_7 \frac{\partial^2 v}{\partial y^2}(i,j)\Delta y^2 + \propto_8 \frac{\partial^3 v}{\partial x^3}(i,j)\Delta x^3 + \propto_9 \frac{\partial^3 v}{\partial x^2\partial y}(i,j)\cdot\Delta x^2\Delta y + + \propto_{10} \frac{\partial^3 v}{\partial x\partial y^2}(i,j)\cdot\Delta x\Delta y^2 + \propto_{11} \frac{\partial^3 v}{\partial x^3}(i,j)\Delta y^3 + + O(\varepsilon^4) \quad (14)$$

where $\propto_2 = g_2 + g_3 + g_4 + g_5 + g_6$,

$$\propto_3 = -g_3 - g_4, \quad (15)$$

$$\propto_4 = -g_2 - g_4 - 2g_5 - 3g_6, \quad (16)$$

$$\propto_5 = \frac{g_3 + g_4}{2}, \quad (17)$$

$$\propto_6 = g_4, \quad (18)$$

$$\propto_7 = \frac{g_2 + g_4}{2} + 2g_5 + \frac{9}{2}g_6, \quad (19)$$

$$\propto_8 = -\frac{g_3 + g_4}{6}, \quad (20)$$

$$\propto_9 = -\frac{1}{2}g_4, \quad (21)$$

$$\propto_{10} = -\frac{1}{2_2}g_4, \quad (22)$$

$$\propto_{11} = -\frac{1}{6}g_2 - \frac{1}{6}g_4 - \frac{4}{3}g_5 - \frac{9}{2}g_6. \quad (23)$$

Therefore, the obtained cubic differential equation (14) is in partial derivatives that approximated a difference scheme (8).

Let us consider in more detail the scheme of the method using the example of a difference scheme for an unknown function of one variable with a maximum distance from the base node in two steps, for example:

$$g_3 v_{k-2} + g_2 v_{k-1} + g_1 v_k + g_0 = 0. \quad (24)$$

Transforming the known difference equation of approximation of the derivative of the second order, the following expression is obtained:

$$\frac{g_3}{h^2}(v_k - 2v_{k-1} + v_{k-2}) = g_3 v''(x_k) + O(h) = 0, \quad (25)$$

where h - discretization step.

Therefore, approximate presentation of the first component of expression is obtained (25):

$$g_3 v_{k-2} = g_3 h^2 v''(x_k) + 2g_3 v_{k-1} - g_3 v_k + O(h^3) \quad (26)$$

Substituting (26) in (24) the next differential-difference equation without value of unknown function in point x_{k-2} is obtained

$$\begin{aligned} &g_3 h^2 v''(x_k) + (g_2 + 2g_3)v_{k-1} + \\ &+(g_1 - g_3)v_k + g_0 + O(h^3) = 0 \end{aligned} \quad (27)$$

Further, using the same scheme only now, using the known difference approximation equation of the derivative of the first order

$$\frac{1}{h}(v_k - v_{k-1}) = v'(x_k) + O(h), \quad (28)$$

we approximate the value of unknown function in the point x_{k-1} by the next way

$$v_{k-1} = -hv'(x_k) + v_k + O(h^2). \quad (29)$$

Substituting (29) in (27) we obtain the next

$$\begin{aligned} &g_3 h^2 v''(x_k) - (g_2 + 2g_3)hv'(x_k) + \\ &+(g_1 + g_2 + g_3)v_k + g_0 + O(h^3) = 0 \end{aligned} \quad (30)$$

Dividing equation (30) into g_3 the next presentation of a differential equation is built

$$h^2 \frac{d^2 v}{dx^2} - h\alpha_1 \frac{dv}{dx} + \alpha_2 v + \alpha_3 + O(h^3) = 0. \quad (31)$$

where

$$\alpha_1 = \frac{g_2 + 2g_3}{g_3}, \quad \alpha_2 = \frac{g_1 + g_2 + g_3}{g_3}, \quad \alpha_3 = \frac{g_0}{g_3} \quad (32)$$

Discarding the higher-order component of smallness in the equation (33), we obtain a second-order inhomogeneous differential equation with a characteristic equation of the form:

$$h^2 k^2 - h\alpha_1 k + \alpha_2 = 0 \quad (33)$$

In case of positive discriminant $D = h^2\alpha_1^2 - 4h^2\alpha_2 > 0$ of characteristic equation, monotonic components of the general solution of exponential type are obtained. Partial solution of a inhomogeneous differential equation (31) will be a constant of the next type $-\frac{\alpha_3}{\alpha_2}$.

Thus, the stated above approach makes it possible to transform difference equation, as a model of a complex object, built on the base of the interval data analysis into a differential equation that interprets the properties of the object, as it describes a well-known phenomenon.

VI. RESULTS

Let us consider the described approach of a transition from the difference equation of the form (24) to a differential equation (31) in the case of such values of the coefficients of the recurrent formula (24):

$$g_0 = -0{,}22278,\ g_1 = 1,\ g_2 = -0{,}728592,\ g_3 = -0{,}133172 \quad (34)$$

These values were obtained as a solution to applied task of modeling the process of spreading carbon monoxide (CO) in the perpendicular direction from the road, as a result of a uniform flow of motor vehicles [21]. Taking into account the values $v(0)$ and $v(h)$ from recurrent presentation of the solution (24) and taking into account the values of coefficients (34), we will obtain the pollution forecast. The obtained results are presented in Table 1. As can be seen, the predicted values based on the model in the form of a difference equation are well consistent with experiment results because forecast intervals belong to intervals obtained by the experiment.

Now let us assess to what extent the solution of the obtained difference equation that is the analog of a difference equation (24) for this problem coincides with the solution in Table 1.

For this purpose, based on the values of the coefficients (34) and the equations (32) the values of the coefficients of approximation differential equation are computed (31):

$$\alpha_1 = \frac{g_2 + 2g_3}{g_3} = 7{,}471,\ \alpha_2 = \frac{g_1 + g_2 + g_3}{g_3} = -1{,}038,$$

$$\alpha_3 = \frac{g_0}{g_3} = 1{,}673, \quad \text{h=10}. \quad (35)$$

Based on these values the discriminant of characteristic equation $D = 59.96884 > 0$ and also real and distinct roots of a characteristic equation $k_1 = -0.01364$, $k_2 = 0.760750756$ are assessed.

Table 1. Measured and predicted values of carbon monoxide

№	Distance from the road, m.	Measured concentration of CO	Interval data concentration of CO	Predicted concentration of CO
k	x_k, m	$\tilde{v}_k, mg/m^3$	$[z_k^-; z_k^+]$ mg/m^3	$[\hat{v}_k]$
0	0	55	[49.5, 60.5]	[52,25;57,75],
1	10	47	[42.3, 51.7]	[44,65;49,35],
2	20	43	[38.7, 47.3]	[39,79;43,09]
3	30	37	[33.3, 40.7]	[34,93;38,12]
4	40	32	[28.8, 35.2]	[30,90; 33,75]
5	50	30	[27.0, 33.0]	[27,96;30,98
6	60	26	[23.4, 28.6]	[24,68;27,13]
7	70	23	[20.7, 25.3]	[21,79;23,68]
8	80	20	[18.0, 22.0]	[19,25;21,02]
9	90	18	[16.2, 19.8]	[17,41;18,54]
10	100	16	[14.4, 17.6]	[15,74;16,6]

The monotonically decreasing component of the general solution of the homogeneous differential equation will correspond to the first root and the monotonically increasing component will correspond to the second root. Based on the values of the solution at the initial observation points 0 and h, the solution of a differential equation should decrease monotonically. Therefore, further, we consider only the component of the solution of the homogeneous differential equation, which corresponds to the first root of the characteristic equation. We determine the constant partial solution of the inhomogeneous differential equation in the form:

$$\overline{v} = -\frac{\alpha_3}{\alpha_2} = 1.6115 \quad (36)$$

This allows recomputing the initial condition to the solution of a homogeneous differential equation and establishing its uncertainty constant. As a result, we obtain a general solution of the inhomogeneous differential equation (31) in the form

$$v(x) = 53.3884e^{-0.01364x} + 1.611599 \quad (37)$$

The pollutant concentration forecast based on the constructed analytical presentation of the solution shown in Table 2.

As it can be seen, the maximum relative error of the forecast for the analytical solution was 2.55%, which is proportionate to the error of the forecast built on the basis of the difference scheme.

Table 2. Measured and predicted by using analytical solution (37) values of carbon monoxide

№	Distance from the road, m.	Measured concentration of CO	Predicted concentration of CO	Abs errors	Rel errors
k	x_k, m	$\tilde{v}_k, mg/m^3$	$v(x)$, mg/m^3	mg/m^3	%
0	0	55	55	0	0
1	10	47	48,1904	-1,190	-2,16
2	20	43	42,2493	0,750	1,36
3	30	37	37,0661	-0,066	-0,12
4	40	32	32,5439	-0,543	-0,99
5	50	30	28,5986	1,401	2,55
6	60	26	25,1564	0,843	1,53
7	70	23	22,1533	0,846	1,54
8	80	20	19,5333	0,466	0,85
9	90	18	17,2474	0,7525	1,37
10	100	16	15,2531	0,7468	1,36

VI. CONCLUSIONS

Therefore, the proposed method gives an opportunity to build the difference equation based on its analogue of the interval differential equation. In the interval differential equation, built on the base of experimental data using the methods of the inductive approach, the coefficients of the difference scheme have no physical meaning, which complicates the interpretation of the physical properties of the object. At the same time, the inductive approach to building mathematical models in the form of the interval difference equation is based on bee colony algorithms and has a sufficient number of modifications implemented by software. Therefore, the proposed method makes it possible to combine the advantages of inductive and deductive approaches to building of mathematical models based on data analysis and to their application and interpretation.

The research results showed that the solutions of both equations (differential and difference) practically coincide. The deviation between the solution of the difference scheme and the solution of the differential equation is 2.55%, which is quite satisfactory for this type of computational problem. This, in turn, creates opportunities for the development of environments for computer modeling of complex objects of various subject areas, since differential equations are universal apparatus for modeling objects and phenomena of various nature. At the same time, the sufficiently unified and effective computational schemes have been developed for the building of interval difference equations, which are based on data analysis. These schemes realize the algorithms to solve the problems of identifying mathematical models as optimization problems.

References

[1] W. Goddard, & S. Melville, *Research Methodology: An Introduction*, 2nd edition, Blackwell Publishing, 2004.

[2] H. R. Bernard, *Research Methods in Anthropology*, 5th edition, AltaMira Press, 2011.

[3] A. G. Ivakhnenko, "Group method of data handling - A rival of the method of stochastic approximation," *Soviet Automatic Control*, no. 13, pp. 43-71, 1966.

[4] M. Saunders, P. Lewis, & A. Thornhill, *Research Methods for Business Students*, 6th edition, Pearson Education Limited, 2012.

[5] W. L. Neuman, *Social Research Methods: Qualitative and Quantitative Approaches*, Allyn and Bacon, 2003.

[6] M. G. Lodico, D. T. Spaulding, & K. H. Voegtle, *Methods in Educational Research: From Theory to Practice*, John Wiley & Sons, 2010.

[7] O. Dag, C. Yozgatligil, "GMDH: An R package for short term forecasting via GMDH type neural network algorithms," *The R Journal*, vol. 8, issue 1, pp. 379-386, 2016. https://doi.org/10.32614/RJ-2016-028.

[8] T. Kondo, J. Ueno, "Revised GMDH-type neural network algorithm with a feedback loop identifying sigmoid function neural network," *International Journal of Innovative Computing, Information and Control*, vol. 2, issue 5, pp. 985-996, 2006. https://doi.org/10.5687/sss.2006.137.

[9] W.-Y. Sun and C.-Z. Chang, "Diffusion model for a convective layer. Part 2: Plume released from a continuous point source," *J. Climate Appl. Meteorol.*, vol. 25, no. 10, pp. 1454-1463, 1986. https://doi.org/10.1175/1520-0450(1986)025<1454:DMFACL>2.0.CO;2.

[10] F. Pasquill, "Atmospheric dispersion parameters in Gaussian plume modeling: [part II. Possible Requirements for Change in the Turner Workbook Values]," EPA-600/4-76-030b, U.S. Environmental Protection Agency, Research Triangle Park, North Carolina, 27711, 1976.

[11] P. S. Venhers′kyy, O. R. Demkovych, "Construction of a mathematical model of the process of liquid filtration in the soil," Bulletin of Lviv university, Series Applied Mathematics Informatics, issue 15, pp. 170-177, 2009. (in Ukrainian)

[12] I. Kopas, *Differential equations*. Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2018, 126 p.

[13] O. Zyubanov, *Differential Equations*, Vasyl' Stus Donetsk National University, 2018, 72 p.

[14] W.-S. Wang "Discrete transform methods of solutions of the fractional difference equations," *Proceedings of the 2018 5th International Conference on Information Science and Control Engineering, ICISCE*, 2019, pp. 714-717. https://doi.org/10.1109/ICISCE.2018.00153.

[15] M. Dyvak, I. Voytyuk, N. Porplytsya and A. Pukas, "Modeling the process of air pollution by harmful emissions from vehicles," *Proceedings of the 2018 14th International Conference on Advanced Trends in Radioelecrtronics, Telecommunications and Computer Engineering (TCSET)*, Slavske, 2018, pp. 1272-1276, https://doi.org/10.1109/TCSET.2018.8336426.

[16] M. Dyvak, N. Porplytsya, "Formation and identification of a model for recurrent laryngeal nerve localization during the surgery on neck organs," *Advances in Intelligent Systems and Computing III. CSIT 2018*,

Cham: Springer, vol. 871, pp. 391-404, 2019. https://doi.org/10.1007/978-3-030-01069-0_28.

[17] Y.-H. Lee, & S. J. Haberman, "Studying score stability with a harmonic regression family: A comparison of three approaches to adjustment of examinee-specific demographic data," *Journal of Educational Measurement*, vol. 58, issue 1, pp. 54–82, 2021. https://doi.org/10.1111/jedm.12266.

[18] A. Agresti, *Categorical Data Analysis*, (second ed.), Wiley, New York, 2002. https://doi.org/10.1002/0471249688.

[19] M. Analla "Model validation through the linear regression fit to actual versus predicted values," *Agricultural Systems*, vol. 57, pp. 115-119, 1998. https://doi.org/10.1016/S0308-521X(97)00073-5.

[20] J. Bibby, H. Toutenburg, *Prediction and Improved Estimation in Linear Models*, Wiley, Berlin, 1977.

[21] M. Dyvak, "Parameters identification method of interval discrete dynamic models of air pollution based on artificial bee colony algorithm," *Proceedings of the 2020 10th IEEE International Conference on Advanced Computer Information Technologies (ACIT)*, Deggendorf, Germany, 2020, pp. 130-135. https://doi.org/10.1109/ACIT49673.2020.9208972..

[22] C. Sun, et al., Stochastic prediction of multi-agent interactions from partial observations, arXiv preprint arXiv:1902.09641, 2019.

[23] J. C. Bansal, P. K. Singh, N. R. Pal, "Evolutionary and swarm intelligence algorithms," Springer Cham, 2019, 190 p. https://doi.org/10.1007/978-3-319-91341-4.

[24] J. Tang, G. Liu and Q. Pan, "A review on representative swarm intelligence algorithms for solving optimization problems: Applications and trends," *IEEE/CAA Journal of Automatica Sinica*, vol. 8, no. 10, pp. 1627-1643, 2021. https://doi.org/10.1109/JAS.2021.1004129.

[25] A. Mortazavi, V. Toğan, M. Moloodpoor, "Solution of structural and mathematical optimization problems using a new hybrid swarm intelligence optimization algorithm," *Advances in Engineering Software*, vol. 127, 2019, pp. 106-123. https://doi.org/10.1016/j.advengsoft.2018.11.004.

[26] H. Wang, W. Wang, X. Zhou, "Artificial bee colony algorithm based on knowledge fusion," *Complex Intell. Syst.*, vol. 7, pp. 1139–1152, 2021. https://doi.org/10.1007/s40747-020-00171-2.

[27] D. Bajer, B. Zorić, "An effective refined artificial bee colony algorithm for numerical optimisation," *Information Sciences*, vol. 504, pp. 221-275, 2019. https://doi.org/10.1016/j.ins.2019.07.022.

[28] K. Hussain, M. N. M. Salleh, Shi Cheng, Yu. Shi, R. Naseem, "Artificial bee colony algorithm: A component-wise analysis using diversity measurement," *Computer and Information Sciences*, vol. 32, issue 7, pp. 794-808, 2020. https://doi.org/10.1016/j.jksuci.2018.09.017.

[29] R. Aguejdad, "The influence of the calibration interval on simulating non-stationary urban growth dynamic using CA-Markov model," *Remote Sensing*, vol. 13, issue 3, pp. 468, 2021. ttps://doi.org/10.3390/rs13030468.

[30] N. T. Thong, L. Q. Dat, L. H. Son, N. D. Hoa, M. Ali, and F. Smarandache, "Dynamic interval valued neutrosophic set: modeling decision making in dynamic environments," *Computers in Industry*, vol. 108, pp. 45–52, 2019. https://doi.org/10.1016/j.compind.2019.02.009.

[31] L. Zhu et al., "Analytic interval prediction of power system dynamic under interval uncertainty," *Journal of Physics: Conference Series*, vol. 2427, 2022, 2nd International Conference on Smart Grid and Energy Internet (SGEI 2022) 21/10/2022 - 23/10/2022 Sanya, China, pp.1-15.

[32] M. J. Simpson, S. A. Walker, E. N. Studerus, S. W. McCue, R. J. Murphy, O. J. Maclaren, "Profile likelihood-based parameter and predictive interval analysis guides model choice for ecological population dynamics," *Mathematical Biosciences*, vol. 355, 108950, 2023. https://doi.org/10.1016/j.mbs.2022.108950.

[33] M. Dyvak, A. Pukas, A., I. Oliynyk, A. Melnyk, "Selection the 'Saturated' block from interval system of linear algebraic equations for recurrent laryngeal nerve identification," *Proceedings of the 2018 IEEE 2nd International Conference on Data Stream Mining and Processing, DSMP'2018*, 2018, pp. 444-448. https://doi.org/10.1109/DSMP.2018.8478528.

[34] X. Jiang, and Z. Bai, "Interval uncertainty quantification for the dynamics of multibody systems combing bivariate Chebyshev polynomials with local mean decomposition," *Mathematics*, vol. 10, no. 12, 1999. https://doi.org/10.3390/math10121999.

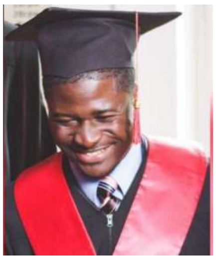
[35] N. R. Kondratenko, O. O. Snihur, "Investigating adequacy of interval type-2 fuzzy models in complex objects identification problems," *System Research and Information Technologies*, no. 4, pp. 94–104, 2019. https://doi.org/10.20535/SRIT.2308-8893.2019.4.10.

***MYKOLA DYVAK**, Dean of Faculty of Computer Information Technologies, Doctor of Technical Sciences, Professor, Honoured Worker of Science and Technology of Ukraine. Scientific Interests: estimation of the states of control objects on the basis of interval approach; mathematical models of the growth of structures.*

***ROMAN PASICHNYK**, a Professor of the Department of Economic Cybernetics and Informatics of the Faculty of Computer Information Technology, Doctor of Technical Sciences, Professor, scientific interests: mathematical models of the growth of structures, information technologies, ontology systems, computer science.*

***Andriy Melnyk**, an Associate Professor of the Department of Computer Science of the Faculty of Computer Information Technology, PhD in Technical Sciences, scientific interests: information technologies, ontology systems, computer science.*

***Andriy Dyvak**, PhD student of West Ukrainian National University. Now he works as a surgeon in the clinical hospital in Ternopil. Research areas: information technologies, medical systems, mathematical models, computer science.*



***Frank Otoo**, PhD student of West Ukrainian National University. Now he works as a assistant in the university of Berlin in Germany.
Scientific interests: mathematical models, computer science.*